第九屆邊界元素法與積分方程及其相關數值方法研討會

The Ninth Workshop on Boundary Element Methods Integral Equations and Related Method in Taiwan

大會手冊與論文摘要集

主辦單位：義守大學財務與計算數學系

協辦單位：國立高雄科技大學造船及海洋工程系
科技部數學研究推動中心
台灣工業與應用數學會

會議時間：2018 年 11 月 10 日

MOST 科技部 Ministry of Science and Technology

TWSIAM 台灣工業與應用數學會 Taiwan Society for Industrial and Applied Mathematics

第九屆 台灣邊界元素法、積分方程及其相關數值方法研討會

The ninth workshop on boundary element and integral equation and related method in Taiwan

日期

2018年 11/10(六) 上午9:00

地點

義守大學綜合教學大樓 50201教室

Keynote Lecturers

演講者	演講題目
陳正宗	Overview of Taiwan BEM series since 1984-paradise and parasite of the BEM/BIEM
楊德良	Applying the Method of Characteristics with the Meshless Localized Radial Basis Function CollocationMethod to Solve Shallow Water Equations
李國明	Dirichlet-to-Neumann maps and their applications in scattering problems
劉立偉	First step of Clifford algebra valued boundary integral equations to 3D elasticity
洪子倫	3D Poisson-Boltzmann computation of ion channel
夏育群	Three-Dimensional BEM Analysis of Generally Anisotropic Thermoelasticity
蔣世中	Numerical Algorithms for Solving Optimal Control Problems with Integro-Differential Equations of the Second Kind as Constraints
李洋傑	Complete multipole series expansion of the fundamental solutions for static micromorphic elasticity
胡潛濱	A special boundary element for coupled stretching-bending analysis of thin laminated plates with elastic inclusions
高仕超	High-performance multi-GPUs finite element solver for the incompressible Navier-Stokes equations
廖駿偉	Optical Force and Torque
胡馨云	A tradeoff between accuracy and efficiency to meshfree weak and strong formulations

Organizer

黃宏財 / 義守大學財務與計算數學系
陳義麟 / 國立高雄科技大學造船及海洋工程系
吳土城 / 義守大學財務與計算數學系

報名資訊

報名截止日為107年10月30日(二)

主辦單位

義守大學財務與計算數學系

協辦單位

科技部數學推動中心
台灣工業與應用數學會
國立高雄科技大學造船及海洋工程學系

線上報名

目錄

研討會背景

邊界元素法(Boundary Element Method, BEM)於 1970 年代發展至今已有 48 年歷史，其主要的理論基礎是由格林恆等式出發所推導得到的積分方程。而邊界元素法這個名詞最早是在 1977 年由 Brebbia 與 Dominguez 發表於 *Applied Mathematical Modelling* (Volume 1, page 372)首先提到。其主要精神則是本著問題的邊界作離散所發展出來的一套數值方法。在國際學術舞台上，邊界元素法一直有許多學者從事這方面的研究，亦有舉辦許多相關學術研討會，如 BETEQ、BETECH、BEM/MRM、IABEM、ICOME、華人工程計算方法學術會議等。此法也被廣泛應用於許多工程問題的分析，如地下水滲流問題、穩態熱傳導分析、裂縫成長分析與預測、大地應力分析等。國外亦有許多軟體公司將此方法發展成套裝軟體，例如: SYSTEM NOISE、BEASY CRACK、BEST3D 等。

在臺灣也曾經針對邊界元素法開設過短期研討課程。1986 年美國Kentucky大學Rizzo教授、Shippy教授及Cornell大學Mukherjee教授在國科會(現在的科技部)的資助下，曾應鮑亦興院士之邀來台講學，於台灣大學應用力學研究所開設為期一週的短期研討課程。事隔 12 年，1998 年國家高速電腦中心再次在科技部的資助下，邀請國內十幾位專家學者，舉辦了一場邊界元素計算研討會。然前述二會並沒有來自數學界的朋友一同參與盛會。之後又過了十二載，國內邊界元素法在這段期間內並未舉辦過相關研討會或研習課程。

在數學領域上，國內也有許多學者對邊界積分方程有所鑽研。然而，在 2010 年之前，國內過去無相關單位或機構針對此相關領域舉辦研討會，提供工程領域與數學領域學者相互切磋與交流的平台。如果能有機會，讓工程師分享其實務應用經驗，而數學家也能提供其研究理論成果，增進彼此雙方交流，進一步達成數學與工程的結合，對學術界而言是很有意義的。基於這些原因，促成了本系列研討會的產生。

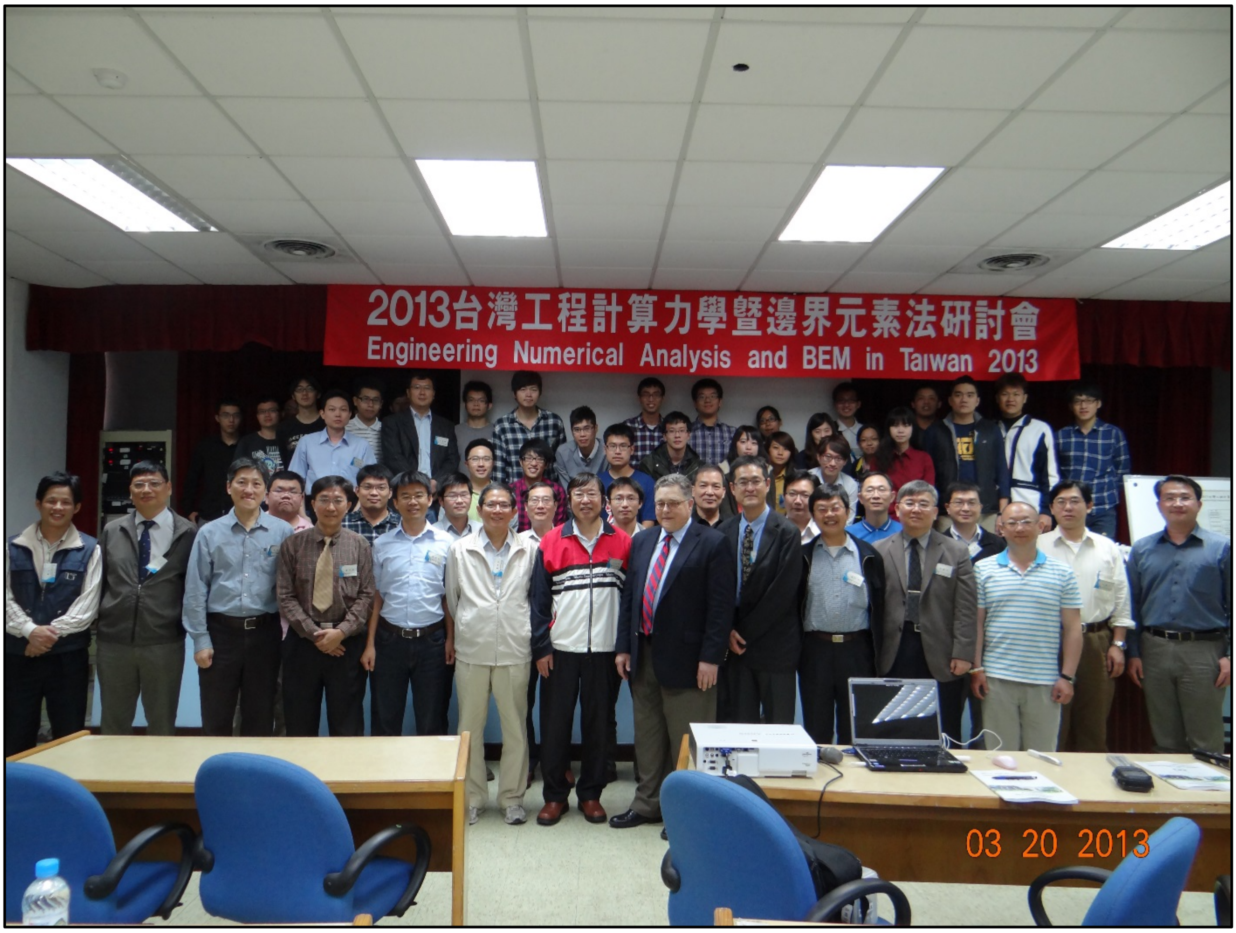
2010 年 10 月 16 日臺灣海洋大學河海工程學系五十週年系慶時陳正宗教授創辦了第一屆邊界元素法與積分方程及其相關數值方法研討會，提供數學與工程學者同好相互切磋的機會。2011 年 3 月 15-18 日中山大學李子才教授、呂宗澤教授、美國密西西比大學程宏達教授、美國南密西西比大學陳清祥教授、台灣大學楊德良教授及臺灣海洋大學陳正宗教授於中山大學舉辦國際研討會『Joint International Workshop on Trefftz Method VI and Method of Fundamental Solution II』；同年成功大學數學系承辦了第二屆邊界元素法與積分方程及其相關數值方法研討會，開啟了此一系列研討會的舉辦(請參閱歷屆照片)。本次第九屆研討會傳承此一薪火，提供一個學術互動平台，使國內工程學者與數學學者相互切磋，交流與學習。感謝中央研究院數學研究推動中心經費補助及台灣工業與應用數學學會的協辦。我們熱情地歡迎對邊界元素法及相關領域有興趣的專家學者們一同前來參與這場盛會，一起來為台灣邊界元素法寫歷史。

第一屆(2010) 海洋大學河海工程學系／陳正宗教授

第二屆(2011) 成功大學數學系／李國明教授

第三屆(2012) 逢甲大學航太系／夏育群教授

第四屆(2013) 中興大學土木工程系／壽克堅教授

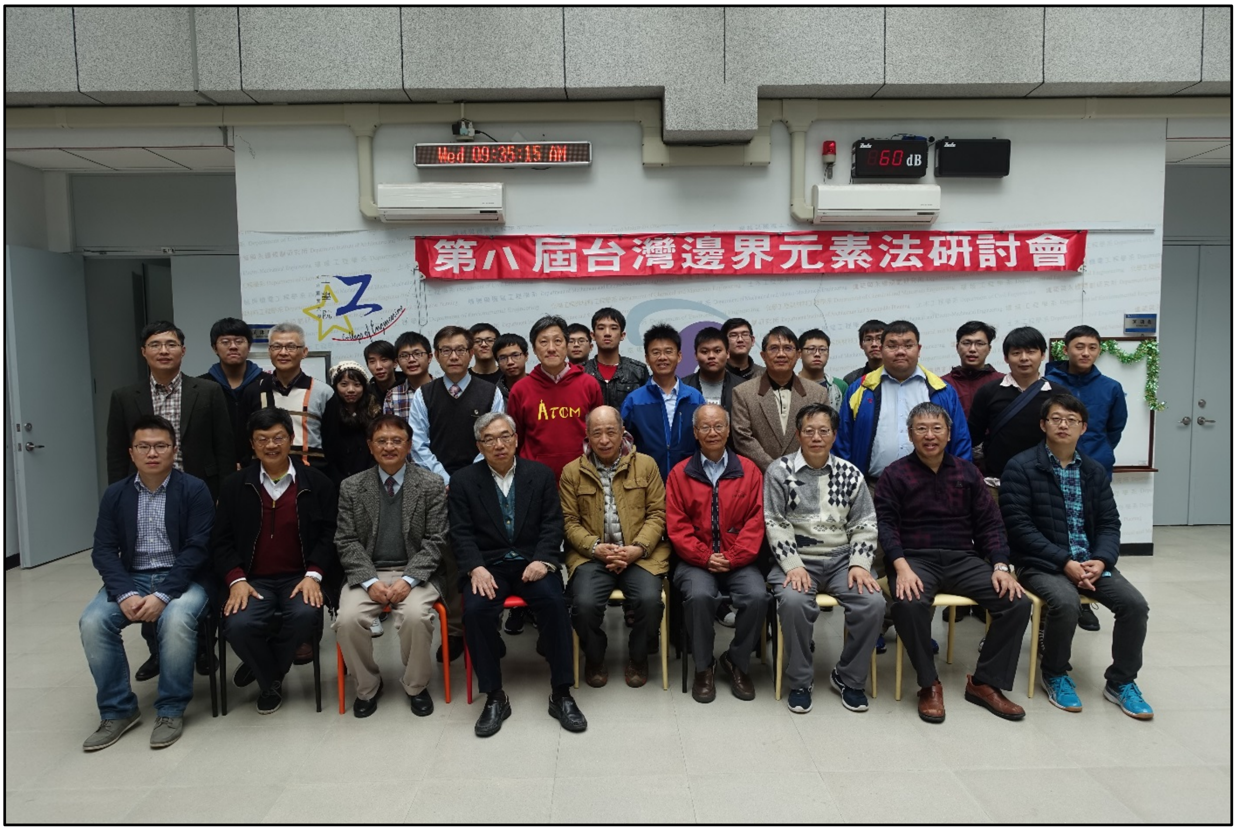


第五屆(2014) 中山大學應用數學系／呂宗澤教授

第六屆(2015) 國家地震中心／王仁佐教授

第七屆(2016) 國家理論科學研究中(NCTS) ／陳宜良教授

第八屆(2017) 宜蘭大學土木工程系／陳桂鴻教授



籌組委員會

主辦單位：義守大學財務與計算數學系

協辦單位：國立高雄科技大學造船及海洋工程系

贊助單位：科技部數學研究推動中心
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主辦人：黃宏財(義守大學財務與計算數學系)

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陳義麟(國立高雄科技大學造船及海洋工程系)
陳正宗(台灣海洋大學河海工程學系)

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郎正廉（義守大學財務與計算數學系）
施俊良（義守大學財務與計算數學系）
羅大欽（義守大學財務與計算數學系）
葉建寧（義守大學財務與計算數學系）
謝良瑜（義守大學財務與計算數學系）
孫一凡（義守大學財務與計算數學系）

義守大學簡介

本校由義联集團創辦人林義守先生，於事業有成後，本著感懷慈母劬養之恩及回饋鄉里之情，秉持為國家培育高級專業人才之理念，於民國 79 年興辦高雄工學院，經過不斷努力，在教學、研究、行政方面都有卓越的表現，於 86 年 8 月經教育部核定升格並更名為「義守大學」。

隨著校務的發展，本校陸續設有電機資訊學院、理工學院、管理學院、國際學院、語文學院、觀光餐旅學院、傳播與設計學院、醫學院及原住民族學院等 9 個學院，除提升學生高科技與管理的專業技術研發核心競爭力外，更進一步培育和醫學、語文、餐旅及文化創意相關之專業人才。95 年醫學相關學系陸續遷入燕巢-醫學院區，目前緊鄰「義大醫院」及「義大癌治療醫院」，使學生獲得更好的學習環境。99 學年度本校獲准設立學士後中醫學系；102 年設立全英語授課學士後醫學系外國學生專班，並歷經十三年的努力，同年成立醫學院，且已獲教育部核准，將於 108 學年度招收醫學系公費生。為培育國際人才，與國際接軌，98 年成立國際學院，並以全英教學打造優質學習環境；另為培育文創產業及服務業之人才，設立傳播與設計學院及觀光餐旅學院，規劃了極佳的職場實習與工讀實習的機會。

本校 102 年通過第二週期系所暨通識教育評鑑，更是唯一同時全數通過的大學；103 年度上半年接受大學校院通識教育及第二週期系所評鑑，新設系所電影與電視學系、數位多媒體設計學系、創意商品設計學系全數通過；103 學年度工程及科技教育認證，本校全數工程學系 29 學制，繼 100 學年度後，再次通過 IEET 認證。此外，本校榮登 2014 年美國發明人協會專利排名，列全球百大創新卓越大學；2015-2017 年連續三年入選英國倫敦《泰晤士報》全球最佳大學及 2016-2018 年連續三年入選英國倫敦《泰晤士報》亞州最佳大學；2016 年管理、國際、觀餐三學院同時通過 AACSB 國際商管教育認證；2017 年首次入列「Round University Ranking 2017 全球最佳大學」之列；2018 年首次進榜《泰晤士報》「全球最佳年輕大學排行榜」、且入選《遠見雜誌》台灣最佳大學排行榜，躍升全國綜合大學第 14 名，為全國年輕大學中表現最佳的一所，贏得「最佳進步獎」殊榮。

107 學年度本校獲高教深耕計畫及大學社會責任實踐(USR)計畫，為展現永續實踐大學社會責任的決心，本校前往屏東長治百合部落執行新年度在地計畫，並舉行「在地計畫執行場域辦公室」揭牌儀式，象徵合作更形緊密強化。本校連續幾年表現亮眼，顯示高品質教育屢獲教育部及國際認證肯定，尤其在教育部 106 年 4 月高教創新電子報公布之國際生能量，本校外國學位生(學士班和碩博班)能量居全國第 3、學士班居全國第 2，展現本校國際化亮麗之成果。

本校為實現建立一所「創新知識、服務社會、永續發展」之國際化綜合大學的願景，以及為因應 2028 年高等教育少子化的衝擊，將推動『ISU 2028 計畫』，規劃以「落實卓越創新教學」、「推動跨域自主學習」、「深化全人素養教育」、「精進四創教育生態」、「厚植前瞻特色研究」、「躍升國際競逐全球」，以及「營造永續幸福校園」等七大校務發展工作，期盼能群策群力、邁向巔峰!

義守大學財務與計算數學系簡介

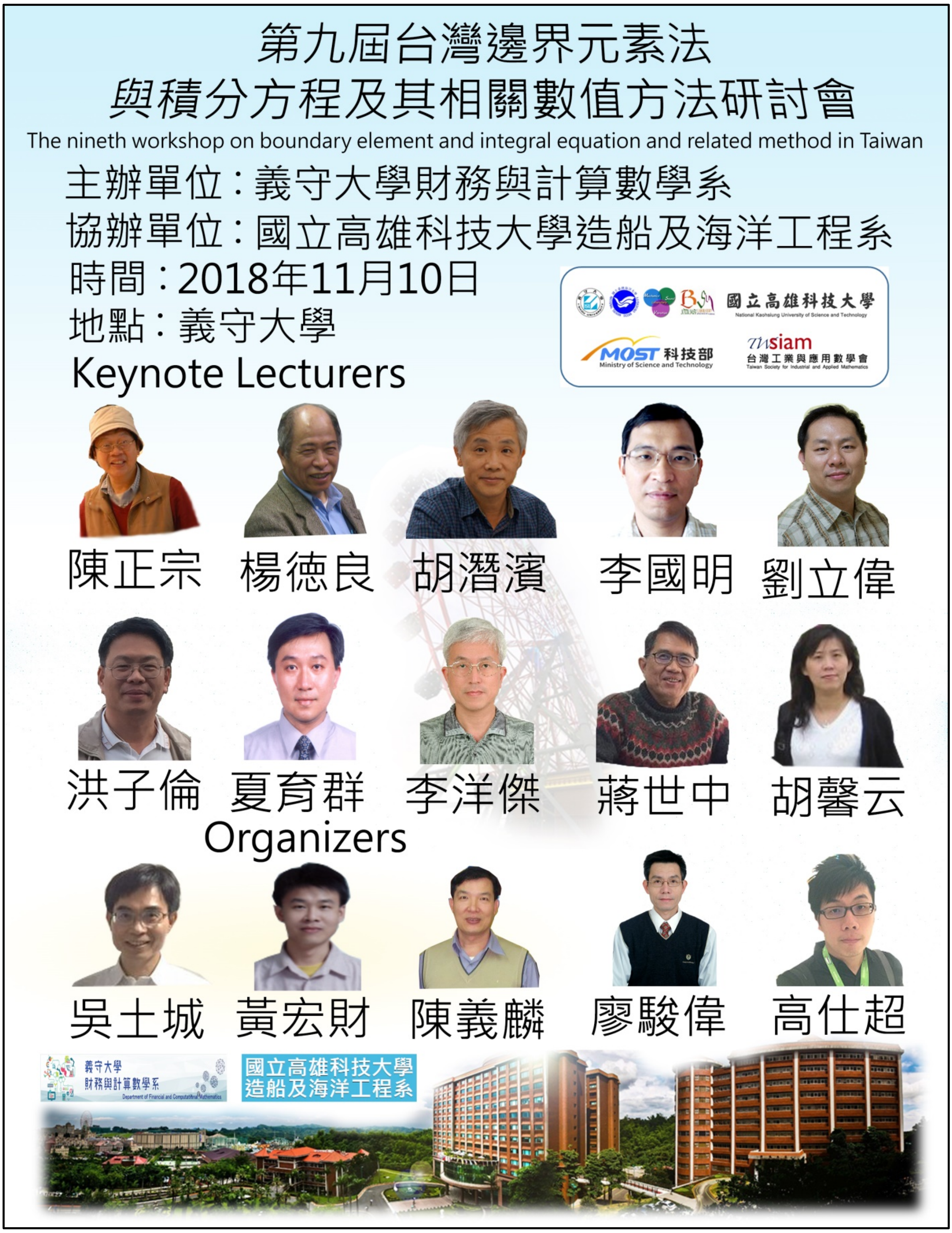
本系創設於民國 79 年，民國 80 年開始招生，民國 104 年更名為財務與計算數學系。目前專任教師十位及合聘教師一位，全數擁有博士學位，皆畢業於為國內外知名大學，其中教授 3 名、副教授 4 名、助理教授 4 名。

數學在各個行業中已成為不可或缺的工具，有著良好的數理能力，將帶來更寬廣的成長空間。本系課程規劃之宗旨在於奠定學生思考、組織、表達及創造之能力，強調基本的數理訓練，「易懂易學、能懂能用」是我們的教學目標。多年來經過業界及學者專家精心設計，融合數理、金融、資訊的特色課程，打破傳統數學窠臼，理論與實務兼備，課程設計著重於數理邏輯、金融科技與資料科學等技能之養成，訓練同學財務金融、程式設計、資料處理及實務操作之專業，培養具備數理分析能力之金融、資訊專業人才。

本系設有「財務分析研究室」與「資訊科學研究室」，提供教學研究之用，在大學系所評鑑中，深受委員肯定，認為本系軟硬體設備完善，甚至優於一流國立大學。本系與產業界結合日益密切，積極參與產學合作，提供業界實習機會，培養學生專業實務能力。推動基礎技能養成，輔導學生取得專業證照，畢業學生在財務、資訊、管理及數學方面的發展，深獲肯定，已有多位系友在學界、業界嶄露頭角，展現優秀的競爭能力。

歷年來，在全體同仁的努力下，本系畢業學生的專業素養已不同於大家傳統對於數學的刻板印象，我們已將學生的專業能力銜接資訊、金融等領域，不僅兼顧數學，並利用其優勢作更好的發展，出路非常廣泛，未來可朝證券、期貨、保險、銀行業、系統工程師、軟體開發設計、多媒體製作、金融科技、大數據/資料科學或教職等產業發展，亦可選擇升學，朝資訊工程、資訊管理、統計、數學、應用數學、工業管理、財務、經濟、數位金融、精算、保險或資料科學等領域精進專業，厚植競爭力。本系有多位畢業生在取得資訊、統計及管理等博士學位後，已應聘至知名的公私立大學任教；另有許多系友在業界已有斐然的成績，未來希望在輔導學生就業上，能拓展更寬廣的版圖。

邀請演講者


第九屆台灣邊界元素法
與積分方程及其相關數值方法研討會
The nineth workshop on boundary element and integral equation and related method in Taiwan
主辦單位：義守大學財務與計算數學系
協辦單位：國立高雄科技大學造船及海洋工程系
時間：2018年11月10日
地點：義守大學
國立高雄科技大學
MOST 科技部
Ministry of Science and Technology
TWSIAM
台灣工業與應用數學會
Keynote Lecturers
陳正宗
楊德良
胡潛濱
李國明
劉立偉
洪子倫
夏育群
李洋傑
蔣世中
胡馨云
Organizers
吳士城
黃宏財
陳義麟
廖駿偉
高仕超
義守大學
財務與計算數學系
國立高雄科技大學
造船及海洋工程系

大會議程

日期：2018 年 11 月 10 日(六)

地點：義守大學綜合教學大樓 50201 教室

時間	主持人	演講者	演講題目
09:00 ~ 09:20			報到
09:20 ~ 09:30	古志生院長		開幕式
09:30 ~ 10:00		陳正宗	Overview of Taiwan BEM series since 1984-paradise and parasite of the BEM/BIEM
10:00 ~ 10:20			大合照
10:20 ~ 10:45			茶點時間
10:45 ~ 11:10	呂宗澤	楊德良	Applying the method of characteristics with the meshless localized radial basis function collocation method to solve shallow water equations
11:10 ~ 11:35		李國明	Dirichlet-to-Neumann maps and their applications in scattering problems
11:35 ~ 12:00		劉立偉	First step of Clifford algebra valued boundary integral equations to 3D elasticity
12:00 ~ 13:30			午餐時間
13:30 ~ 13:55	李明恭	洪子倫	3D Poisson-Boltzmann computation of ion channel
13:55 ~ 14:20		夏育群	Three-dimensional BEM analysis of generally anisotropic thermoelasticity
14:20 ~ 14:45		蔣世中	Numerical algorithms for solving optimal control problems with integro-differential equations of the second kind as constraints
14:45 ~ 15:10		李洋傑	Complete multipole series expansion of the fundamental solutions for static micromorphic elasticity
15:10 ~ 15:40			茶點時間
15:40 ~ 16:05	陳義麟	胡潛濱	A special boundary element for coupled stretching-bending analysis of thin laminated plates with elastic inclusions
16:05 ~ 16:30		高仕超	High-performance multi-GPUs finite element solver for the incompressible Navier-Stokes equations
16:30 ~ 16:55		廖駿偉	Optical force and torque
16:55 ~ 17:20		胡馨云	A tradeoff between accuracy and efficiency to meshfree weak and strong formulations
17:20 ~ 17:30			閉幕式
17:30 ~ 20:30			晚餐時間

Overview of Taiwan BEM series since 1984-paradise and parasite of the BEM/BIEM

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Abstract

It is well known that BEM/BIEM is an acceptable approach for solving engineering problems. It is a paradise in several aspects, one-dimension reduction of mesh, infinite domain, stress concentration and crack problems. Several successful experiences in Taiwan will be demonstrated. However, it also results in parasites in some cases. Rank-deficient matrix appears due to the degenerate scale, degenerate boundary, spurious eigenvalue and fictitious frequency once the BIEM/BEM is used for solving boundary value problems. Based on the well-posed dual formulation of the BEM/BIEM, full rank promotion can be achieved. In this talk, three parts will be given. First, the TwSIAM will be introduced. Second, the history of Taiwan BEM development will be addressed. Third, the rank-deficiency system in the BIEM/BEM will be reviewed and transformed to the well-posed system. Finally, I will mention recent works of NTOU/MSV group on the quaternion and the Clifford BEM.

Keywords: rank deficiency, degenerate scale, BEM/BIEM

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Applying the Method of Characteristics with the Meshless Localized Radial Basis Function Collocation Method to Solve Shallow Water Equations

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Abstract

This paper proposes an accurate and efficient numerical model by combining the method of characteristics (MOC) and the meshless localized radial basis function collocation method (LRBFCM) to simulate the shallow water flow problems. The shallow water equations (SWEs) are classified into a hyperbolic-type partial differential equations (PDEs) system that easily creates numerically unstable results for the case with discontinuous field values or shock waves. To solve this problem, the SWEs are derived into conservative eigensystem form, and then the MOC is applied to capture the change of conservative variables along the characteristic lines. Specifically, the meshless LRBFCM is used to obtain the field values from the conservative variables; it can ease the complexity of the interpolation procedure on characteristic Lagrangian points and preserve the accuracy in transient problems. For the boundary disposal, a fractional time step skill with the characteristic velocity is considered to determine the boundary requirements. The computational nodes can be generated by the uniform or nonuniform distribution, which reduces the difficulty of node generation to obtain efficient and accurate numerical analysis. Six continuous and discontinuous SWEs benchmark examples are simulated and discussed to verify the proposed model. The excellent agreements with the analytical, experimental, and numerical solutions demonstrate the accuracy and efficiency of the algorithm.

Dirichlet-to-Neumann maps and their applications in scattering problems

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Abstract

A Dirichlet-to-Neumann (DtN) map, as its name suggests, is an operator which maps the Dirichlet data of a function to its Neumann data. In this talk, we consider DtN maps in form of boundary integral operators arising from some physical problems. As for the underlying applications, we choose two problems from the scattering theory: a transmission problem and a source problem. We define DtN maps for each problem and illustrate the solution process for these problems through some numerical examples.

First step of Clifford algebra valued boundary integral equations to 3D elasticity

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Abstract

Applications of Clifford algebra and Clifford analysis to three-dimensional elasticity will be introduced in this talk. Firstly, the governing equation of isotropic elasticity in three-dimensional space for the displacement field is formulated in terms of the Dirac operator and Clifford algebra valued functions so that a general solution is obtained analytically in terms of one monogenic function and one multiple-component spatial harmonic function together with its derivative. Furthermore, Clifford algebra valued boundary integral equations (BIEs) for multiple-component spatial harmonic functions at an observation point, either inside the domain, on the boundary, or outside the domain, are constructed. Both smooth and non-smooth boundaries are considered in the construction. Moreover, the singularities of the integrals are evaluated exactly so that in the end singularity-free BIEs for the observation point on the boundary taking values on Clifford numbers can be obtained. A Clifford algebra valued boundary element method (BEM) based on the singularity-free BIEs is then developed for solving three-dimensional problems of isotropic elasticity. The accuracy of the Clifford algebra valued BEM is demonstrated numerically. On the other hand, the governing equations of three dimensional elasticity of anisotropic materials is converted into eigenvalue problems taking values of Clifford numbers using the coordinate transformation as what is done in the Lekhnitskii formalism or the Stroh formalism for two-dimensional problems. After solving the eigenvalue problems with the aid of Clifford analysis, we obtain a three-dimensional general solution of anisotropic elasticity.

3D Poisson-Boltzmann computation of ion channel

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Abstract

Ion channels are pore-forming trans-membrane proteins that allow ions to enter/leave cell. There are many important cell functions involving ion channel, e.g., establishing and regulating action potential in neurons and myocytes. The average time for an ion passing through ion channel is in the order of ms, which is infeasible for molecular dynamics simulation so far. Continuum model like Poisson-Boltzmann equation (PB) and Poisson-Nernst-Planck (PNP) equations are popular to describe ion channel in equilibrium and non-equilibrium situations. KcsA potassium channel is chosen to be studied here, since it is one of few ion channels having X-ray crystallographic structure. 3D PB and PNP simulations of KcsA channel have been a challenging task, since (1) geometry is complicated especially the narrow filter part requiring high resolution when generating meshes; (2) mathematical models are complicated since there are various versions of modified PB/PNP to choose; (3) physics is complicated such as distributions of dielectric constant and diffusion coefficient, necessity to employ steric effect or not and solvation energy should be included or not. Here, a PDB 3F7Y KcsA structure with filter part replaced by that of PDB 1K4C is used as the structure for simulation. Unlike all other KcsA PDB structures, this synthetic structure guarantees that the channel is open. PB and modified PB equations are first extended to be pseudo-time-dependent with the steady-state solution being our only interest. The numerical framework adopted here to solve these time-dependent equations for electric potential is method of lines (MOL). Governing equation is first semi-discretized in space by 2ndorder finite volume method under Cartesian grids with the edge value to cope with interface condition. This semi-discretized system forms a system of ordinary differential algebraic equations (ODAE) that can be further integrated by popular ODAE solvers. Mathematical models simulated here are (I) Classical PB, (II) modified PB with steric effect described by Bikerman model, and (III) modified PB as (II) with solvation energy included in addition. From simulation results, we found potassium ion is unrealistically crowded in the filter for model (I). For model (II), though potassium ion is no more unrealistically crowded in filter due to the inclusion of steric effect, there is no room for water to be in the filter. Finally, model (III) delivers the most reasonable physical result among all three models by obtaining reasonable potassium concentration under steric effect and allowing water residence in the filter at the same time.

Three-Dimensional BEM Analysis of Generally Anisotropic Thermoelasticity

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Abstract

As is well known in the BEM community, thermal effect reveals itself as an additional volume integral, whose direct evaluation will generally require domain discretization, destroying the BEM's most distinctive notion of boundary discretization. For treating 3D anisotropic thermoelasticity when no heat source is involved, the leading author previously presented a volume-to-surface integral transformation that relies on a technique of domain mapping using coordinate transformation. In engineering practice, it is quite often to have non-uniform volume heat sources present inside domain that can be modelled using a quadratic function via any statistical means. For treating 3D anisotropic thermoelasticity involving quadratic volume heat sources, this article presents a direct transformation of the volume integral that does not rely on any domain distortion like before. In this article, the directly transformed boundary integral equation is applied to analyze interfacial stresses in 3D dissimilarly adjoined anisotropic bodies subjected to thermal loads.

Keywords: Three-dimensional anisotropic thermoelasticity, direct volume integral transformation, boundary element method (BEM), quadratic volume heat source.

Numerical Algorithms for Solving Optimal Control Problems with Integro-Differential Equations of the Second Kind as Constraints

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Abstract

This study presents numerical algorithms for solving optimal control problems with a class of integro-differential equations of the second kind as costraints. This class of equations consists of an integro-differential term containing an Abel-type kernel. The first kind equations, with a weakly singular kernel, investigated here appear in the mathematical model of an aeroelasticity problem [1]. Two controls are considered in this study: delay and stochastic. The feasibility of the proposed numerical algorithm is demonstrated with examples in which the costs are compared with deterministic optimal controls without time lag.

Keywords: numerical algorithms, integro-differential equations of the second kind, Abel-type kerne

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Complete multipole series expansion of the fundamental solutions for static micromorphic elasticity

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摘要

眾所週知，對於傳統彈力(三個分量之位移向量為基本運動學參數)，Love爵士在20世紀初已經提出線性等向性靜彈之三維并矢格林函數的因子分解多極展開(multipole expansion)解析形式表示式，其組成因子為兩種型態各三組系列的各對應項乘積之加總所構成三個無限級數：其中一種型態為，成原點為奇異場之具有完整性的多極無限展開式解析形式通解。假使R_r為場點到原點距離，而R_s為源點到原點距離，則$R_> = \max(R_r, R_s)$，$R_< = \min(R_r, R_s)$。在1964年R.D. Mindlin 提出Microstructure elasticity模型(Micro-structure inlinear elasticity, Archive for Rational Mechanics and Analysis, 1964)，幾乎同時A.C. Eringen提出Microstructure elasticity模型(Nonlinear theory of simple micro-elastic solids, I & II, International Journal of Engineering Science, 1964)，這兩種模型在巨觀位移向量(以下簡稱$\mathbf{u}$)之外增加一個微觀變形二階張量(以下簡稱ϕ)做為基本運動學參數，因此每一物質點總共有12個自由度。兩人所提出的模型可以証明為等效的，以下統稱為Microstructure elasticity模型。在線性、等向性及非手徵(non-chiral, centrosysmmetric)的假設下，材料常數共有18個。由於這個模型太過複雜，五十多年來可以做為基準解的解析解在前人文獻中非常稀少，據本文作者所知三維Microstructure elasticity靜力解析形式解只可以找到1966年 J.L. Bleustein 所發表就"球對稱應力作用下的球形孔穴的應力集中"之完全球對稱問題所發表的解析形式解 (Effects of micro-structure on the stress concentration at a spherical cavity, International Journal of Solids and Structures, 1966)。A.C. Eringen 在他於1999出版的"Microcontinuum Field Theiries, I. Foundations and Solids"一書中，對此一模型做出以下的斷言："In spite of its age, since its inception in 1964,micromorphic mechanics remains a field that is not cultivated. Its potential is yet to be discovered by research workers. This is due to the complexity of its field equations. Nevertheless, this field of mechanics offers great potential in dealing with deformable microstructure." 為回應這樣的期待，藉由引入量子角動力學的數學，終於完成2×2矩陣三維micromorphic靜彈二～四階張量格林函數($\mathbf{G}_{\mathbf{u},\mathbf{u}}$二階張量、$\mathbf{G}_{\mathbf{u},\emptyset}$三階張量、$\mathbf{G}_{\emptyset,\mathbf{u}}$三階張量、$\mathbf{G}_{\emptyset,\emptyset}$四階張量)的因子分解多極展開解析形式表示式。這一矩陣的組成因子為兩種型態各十二組系列的各對應項矩陣乘積展開之加總所構成十二個無限級數：其中一種型態為原點處規則、無限遠處奇異的位移向量及微觀變形張量、$R_<$相關之"規則Microstructure elasticity多極基本解"，另外一種型態為原點奇異、無限遠處規則的位移向量及微觀變形張量、$R_>$相關之"奇異 Microstructure elasticity多極基本解"。基於Microstructure elasticity 推廣之Somigliana 公式，前一種型態十二組基本解構成原點為規則場之具有完整性的多極無限展開式解析形式通解；另一種型態十

二組基本解構成原點為奇異場之具有完整性的多極無限展開式解析形式通解。鑑於前述所描述的Love的解析形式級數解提供許多傳統靜彈研究者做出各種傳統靜彈的解析形式基準解(Benchmark solutions)，可以期待在此所提出的這些解析形式通解很有潛力力讓後續的研究者創造出各種的 Microstructure elasticity 之解析形式基準解。更進一步，我們希望有更多人投入Microstructure elasticity 的研究，讓這個模型作為處理可變形微觀結構的潛力能得到充分的展現。

A Special Boundary Element for Coupled Stretching-Bending Analysis of Thin Laminated Plates with Elastic Inclusions

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Abstract

For unsymmetric laminates, due to the asymmetry of material properties about mid-plane, the coupling in stretching and bending will occur, which would complicate the analysis. Neither the methods of analyses for pure stretching, nor those for pure bending, are feasible for unsymmetric laminates. Fortunately, the coupled deformation can be analyzed by the Stroh-like formalism developed about 15 years ago [1]. Owing to the identical mathematical form with Stroh formalism for two-dimensional linear anisotropic elasticity, most of the techniques and identities developed for the two-dimensional problems can be employed in Stroh-like formalism. With such advantage, several Green's functions for unsymmetric laminates have been derived. On the other hand, for the coupled deformation, the boundary element method has also been successfully implemented by using the fundamental solution from the Green's function for the infinite unsymmetric laminates [2]. With this achievement, now we would like to develop a special boundary element by employing the Green's function for the infinite unsymmetric laminates with elastic inclusions. Actually, such Green's function has been derived about one decade ago [3]. However, after numerical computation, we found there would be unexpected discontinuities for some physical quantities calculated from the existing solutions, which violates the assumption of continuum model and has not been discovered in the literature. Almost the discontinuities are induced by the arbitrary neglect of constant terms treated as rigid motion. If the corresponding constant values are different for matrix and inclusion, the gap of displacements across the interface would occur. While the other discontinuities come from the improper evaluation of the multi-valued complex logarithmic functions [4]. Additionally, only the in-plane displacements and rotation angles with respect to $x1$- or $x2$-axis are directly provided in Stroh-like formalism, therefore, the integral via rotation angles is necessary to calculate the deflection. Even though the integration can be done analytically or numerically, some integral constants must be treated deliberately to obtain the continuous deflection. After the appropriate rectifying for the Green's function in [3], now we can convert this Green's function into the fundamental solution required by boundary element method for coupled stretching-bending deformation. Then the analyses for unsymmetric laminates with inclusions can be accomplished and is demonstrated by numerical examples, which is expected to be much more efficient and accurate than the conventional boundary element.

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High-performance multi-GPUs finite element solver for the incompressible Navier-Stokes equations

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Abstract

In this study, a multi-GPU-based finite element flow solver is developed to solve the incompressible Navier-Stokes equations. To circumvent the convection instability problem at high Reynolds numbers. the proposed streamline upwind finite element model minimized the wavenumber error for the convection term. Mixed finite element formulation is adopted and the resulting nearly ill-conditioned finite element equations are solved iteratively. To avoid the Lanczos or pivoting breakdown, the finite element equations are first normalized. The computationally efficient preconditioned conjugate gradient (PCG) solver can then be applied to get the unconditionally convergent solution. The developed finite element code implemented on multi-GPU cards will be verified and validated by solving the problem amenable to analytical solution and the benchmark lid-driven cavity problem, respectively. The resulting speedup demonstrates that the application of GPU computing is a worthwhile undertaking.

Keywords: CUDA, Finite element, Multi-GPU, PCG, Incompressible

Optical Force and Torque

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Abstract

The optomechanical trapping performances (optical force and torque) of plasmonic nanostructures upon a dielectric nanoparticle (NP) in a vicinity are studied theoretically. First, the multiple multipole (MMP) method, which is a meshless method, is employed to simulate the electromagnetic (EM) field induced by the plasmonic nanostructures interacting with a linearly polarized (LP) or circularly polarized (CP) Gaussian beam [1-11]. In terms of the EM field, the surface integration of Maxwell's stress tensor on the NP is calculated to analyze the optical force and torque. Through the streamline of the optical force, we investigate the trapping performance of the nanostructure upon the NP. For example, the optical force on a dielectric NP located at different position provided by a 1D gold dimer array irradiated by a LP Gaussian beam is simulated. Our results demonstrate that the trapping mode of the dimer array can be contact or noncontact. In particular, the latter is related to the optical vortex trapping; the trapped NP floats in the medium without touching the nanostructure. The numerical results are in agreement with the experimental ones [12].

Keywords: MMP, optomechanic, electromagnetic field, light-matter interaction, Maxwell's stress tensor, optical force, optical torque, optical vortex, plasmonic nanostructure

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A tradeoff between accuracy and efficiency to meshfree weak and strong formulations

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Abstract

In Galerkin- and collocation-based meshfree methods, an increased number of source points increases accuracy. Therefore, a tradeoff exists between the two. For Galerkin methods, the total number of quadrature points required to obtain optimal solution is high, direct domain integration or variationally consistent integration can achieve optimal convergence with a fewer number of quadrature points. On the other hand, collocation method does not have such a requirement, but requires a sufficient number collocation points for optimal convergence, as well as higher order derivatives of the RK approximation functions. In this work, the Galerkin- and collocation-based meshfree methods are compared in terms of CPU times and errors are evaluated theoretically, and strategies will be discussed.

Keywords: reproducing kernel particle method, reproducing kernel collocation method, Gaussian quadrature, variationally consistent integration, complexit