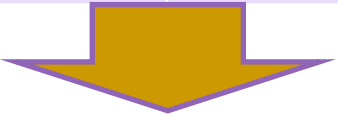


BEM highly-cited papers

(June 14, 2013)

Time	Author	Journal	Cited No.	Method
1967	Rizzo	QAM	401	BEM
1976	Crouch	IJNME	209	DDM
1988	Hong & Chen	ASCE-EM	128	Dual BEM
				
1992	Portela & Aliabadi	IJNME	317	Dual BEM
1997	Mukherjee couple	IJNME	183	BNM
1999	Chen & Hong	ASME-AMR	174	Dual BEM
2002	Nishimura	ASME-AMR	160	FMM BEM

THE DUAL BOUNDARY ELEMENT METHOD: EFFECTIVE IMPLEMENTATION FOR CRACK PROBLEMS

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SUMMARY

The present paper is concerned with the effective numerical implementation of the two-dimensional dual boundary element method, for linear elastic crack problems. The dual equations of the method are the displacement and the traction boundary integral equations. When the displacement equation is applied on one of the crack surfaces and the traction equation on the other, general mixed-mode crack problems can be solved with a single-region formulation. Both crack surfaces are discretized with discontinuous quadratic boundary elements; this strategy not only automatically satisfies the necessary conditions for the existence of the finite-part integrals, which occur naturally, but also circumvents the problem of collocation at crack tips, crack kinks and crack-edge corners. Examples of geometries with edge, and embedded crack are analysed with the present method. Highly accurate results are obtained, when the stress intensity factor is evaluated with the J -integral technique. The accuracy and efficiency of the implementation described herein make this formulation ideal for the study of crack growth problems under mixed-mode conditions.

INTRODUCTION

The boundary element method (BEM) is a well established numerical technique in the engineering community, see Brebbia and Dominguez.¹ Its formulation in elastostatics can be based either on Betti's reciprocity theorem,² or simply based on the classical work theorem.³ In both cases, a single boundary integral equation is obtained. The BEM has been successfully applied to linear elastic problems in domains containing no degenerated geometries. These degeneracies, either internal or edge surfaces which include no area or volume and across which the displacement field is discontinuous, are defined as mathematical cracks. For symmetric crack problems only one side of the crack need be modelled and a single-region BEM analysis may be used. However, in a single-region analysis, the solution of general crack problems cannot be achieved with the direct application of the BEM, because the coincidence of the crack surfaces gives rise to a singular system of algebraic equations. The equations for a point located at one of the surfaces of the crack are identical to those equations for the point, with the same co-ordinates, but on the opposite surface, because the same integral equation is collocated with the same integration path, at both coincident points.

Some special techniques have been devised to overcome this difficulty. Among these the most important are: the crack Green's function method,⁴ the displacement discontinuity method,⁵ the subregions method,⁶ and the dual boundary element method.⁷ The crack Green's function

addition, the discontinuous elements circumvent the problem of collocation at crack tips, crack kinks and crack-edge corners. The effective treatment of the hypersingular integrals that appear in the traction equation is of fundamental importance. For curved boundary elements a regularization integration formula, based on the definition of ordinary finite-part integrals, is proposed in the present paper. For flat boundary elements, the direct analytic integration is the most effective method to deal with such integrations. At a crack node, singular integrations do occur twice, once on the self-point element and again in the opposite one. This feature prevents the use of the standard rigid body condition to evaluate indirectly the diagonal terms of the algebraic equations at crack nodes. Several cracked geometries were analysed with the DBEM; accurate stress intensity factors were always obtained with the J -integral method.

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