Fourier series

海大河海系　　陳正宗

Fourier series expansion :

Orthogonal sets: $\{1, cos(nt), sin(nt)\}$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(nt) + b_n sin(nt)$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} c_n cos(nt + \theta_n)$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} c_n cos(nt - \delta_n)$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} c_n sin(nt + \alpha_n)$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} c_n sin(nt - \beta_n)$$

where

$$c_n^2 = a_n^2 + b_n^2$$

$$\theta_n = tan^{-1} \frac{b_n}{a_n}$$

Orthogonal property for real bases:

$$\int_0^{2\pi} cos(nt)cos(mt)dt = \pi\delta_{mn}$$

$$\int_0^{2\pi} sin(nt)sin(mt)dt = \pi\delta_{mn}$$

$$\int_0^{2\pi} cos(nt)sin(mt)dt = 0$$

Complex Fourier series expansion :

Orthogonal sets: $\{e^{int}\}$

$$f(t) = \sum_{n=-\infty}^{\infty} d_n e^{int}$$

where

$$d_{-n} = \frac{1}{2}\{a_n + ib_n\}, n = 1, 2, 3, \cdots$$

$$d_n = \frac{1}{2}\{a_n - ib_n\}, n = 1, 2, 3, \cdots$$

$$d_0 = a_0$$

Orthogonal property for complex bases:

$$\int_0^{2\pi} e^{int}(e^{imt})^* dt = 2\pi\delta_{mn}$$

where $*$ denotes the complex conjugate.

Complex conjugate pair: (e^{int}, e^{-int}) and (d_n, d_{-n})