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Fourier sum (point)

系級:__________ 學號:_____

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$$f(t)=a_0+\sum_{n=1}^{\infty}(a_n\cos(\frac{n\pi t}{p})+b_n\sin(\frac{n\pi t}{p}))=\frac{1}{4}+\sum_{n=1}^{\infty}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cos(n\pi t)-\frac{\cos(n\pi)}{n\pi}\sin(n\pi t))$$

試求(1) $t=0$ (2) $t=\frac{1}{2}$ (3) $t=1$時， $f(t)=$ ？($n=1\sim5$)

$f(t)=f(t+2)$

$f(t)=0,\ -1<t<0,$

$f(t)=t,\quad 0<t<1$

<sol>

$$f(0)=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cos(n\pi\cdot 0)-\frac{\cos(n\pi)}{n\pi}\sin(n\pi\cdot 0))$$

$$=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cdot 1-\frac{\cos(n\pi)}{n\pi}\cdot 0)$$

$$=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1))$$

$$=\frac{1}{4}+\frac{1}{\pi^2}(\cos(\pi)-1)+\frac{1}{4\pi^2}(\cos(2\pi)-1)+$$

$$\frac{1}{9\pi^2}(\cos(3\pi)-1)+\frac{1}{16\pi^2}(\cos(4\pi)-1)+\frac{1}{25\pi^2}(\cos(5\pi)-1)$$

$$=\frac{1}{4}+\frac{1}{\pi^2}(-1-1)+\frac{1}{4\pi^2}(1-1)+\frac{1}{9\pi^2}(-1-1)+\frac{1}{16\pi^2}(1-1)+\frac{1}{25\pi^2}(-1-1)$$

$$=\frac{1}{4}-\frac{2}{\pi^2}-\frac{2}{9\pi^2}-\frac{2}{25\pi^2}=0.0167$$

$$f(\frac{1}{2})=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cos(n\pi\cdot\frac{1}{2})-\frac{\cos(n\pi)}{n\pi}\sin(n\pi\cdot\frac{1}{2}))$$

$$=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cdot 0-\frac{\cos(n\pi)}{n\pi}\cdot\sin(\frac{n}{2}\pi))$$

$$=\frac{1}{4}+\sum_{n=1}^{5}(-\frac{\cos(n\pi)}{n\pi}\cdot\sin(\frac{n}{2}\pi))$$

$$=\frac{1}{4}-\frac{\cos(\pi)}{\pi}\cdot\sin(\frac{1}{2}\pi)-\frac{\cos(2\pi)}{2\pi}\cdot\sin(\pi)-$$

$$\frac{\cos(3\pi)}{3\pi}\cdot\sin(\frac{3}{2}\pi)-\frac{\cos(4\pi)}{4\pi}\cdot\sin(2\pi)-\frac{\cos(5\pi)}{5\pi}\cdot\sin(\frac{5}{2}\pi)$$

$$=\frac{1}{4}-\frac{-1}{\pi}\cdot 1-\frac{-1}{3\pi}\cdot(-1)-\frac{-1}{5\pi}\cdot 1=\frac{1}{4}+\frac{1}{\pi}-\frac{1}{3\pi}+\frac{1}{5\pi}$$ =0.5259

$$f(1)=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cos(n\pi\cdot 1)-\frac{\cos(n\pi)}{n\pi}\sin(n\pi\cdot 1))$$

$$=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cdot\cos(n\pi)-\frac{\cos(n\pi)}{n\pi}\cdot 0)$$

$$=\frac{1}{4}+\sum_{n=1}^{5}(\frac{1}{n^2\pi^2}(\cos(n\pi)-1)\cdot\cos(n\pi))$$

$$=\frac{1}{4}+\frac{1}{\pi^2}(\cos(\pi)-1)\cdot\cos(\pi)+\frac{1}{4\pi^2}(\cos(2\pi)-1)\cdot\cos(2\pi)+$$

$$\frac{1}{9\pi^2}(\cos(3\pi)-1)\cdot\cos(3\pi)+\frac{1}{16\pi^2}(\cos(4\pi)-1)\cdot\cos(4\pi)+$$

$$\frac{1}{25\pi^2}(\cos(5\pi)-1)\cdot\cos(5\pi)$$

$$=\frac{1}{4}+\frac{1}{\pi^2}(-1-1)\cdot(-1)+\frac{1}{4\pi^2}(1-1)\cdot 1+\frac{1}{9\pi^2}(-1-1)\cdot(-1)+$$

$$\frac{1}{16\pi^2}(1-1)\cdot 1+\frac{1}{25\pi^2}(-1-1)\cdot(-1)$$

$$=\frac{1}{4}+\frac{2}{\pi^2}+\frac{2}{9\pi^2}+\frac{2}{25\pi^2}$$ =0.4833