

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\pi x) + b_n \sin(n\pi x)]$$

$$a_0 = \frac{1}{2L} \int_0^{2L} f(x) dx = \frac{1}{2}$$

$$a_n = \frac{1}{L} \int_0^{2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx = \frac{-2(-1+\cos(n\pi))}{n^2\pi^2}$$

$$b_n = \frac{1}{L} \int_0^{2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx = \frac{2(n\pi - \sin(n\pi))}{n^2\pi^2} = \frac{2}{n\pi}$$

$$f(0) = 1 \text{ (converge in the mean)}$$

$$= 1/2 + \sum_{n=1}^{\infty} \frac{-2(-1+\cos(n\pi))}{n^2\pi^2}$$

$$1/2 = \sum_{n=1}^{\infty} \frac{-2(-1+(-1)^n)}{n^2\pi^2} = \sum_{n=1}^{\infty} \frac{-2(-1+(-1)^n)}{n^2\pi^2} = \sum_{k=0}^{\infty} \frac{4}{(2k+1)^2\pi^2}$$

$$\pi^2/2 = \sum_{n=1}^{\infty} \frac{-2(-1+(-1)^n)}{n^2\pi^2} = \sum_{n=1}^{\infty} \frac{-2(-1+(-1)^n)}{n^2\pi^2} = \sum_{k=0}^{\infty} \frac{4}{(2k+1)^2}$$

