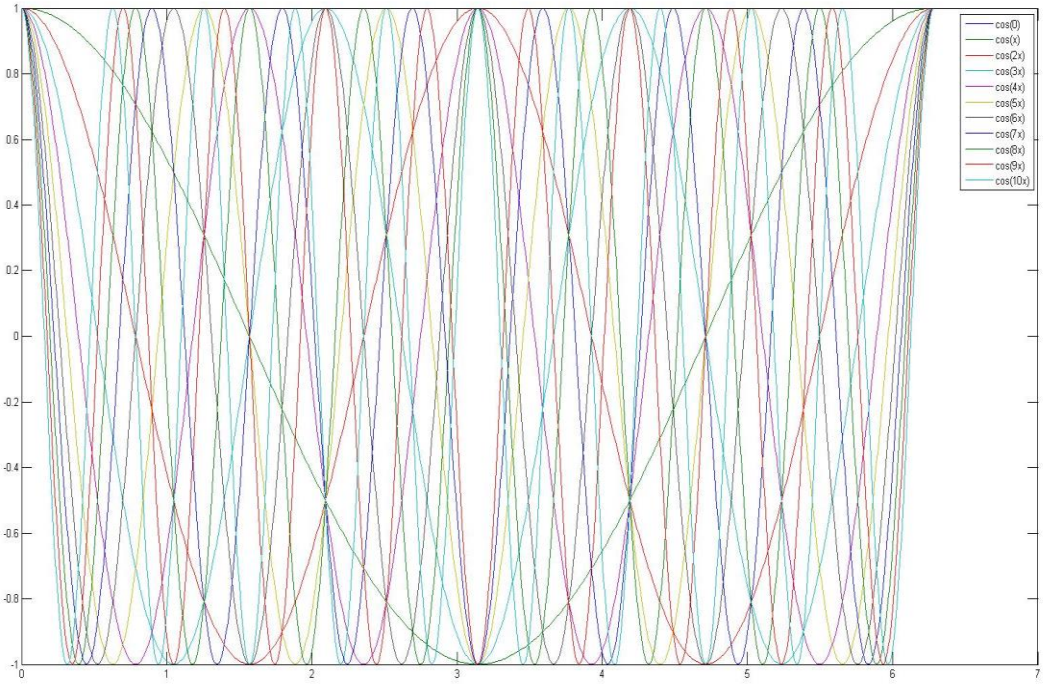
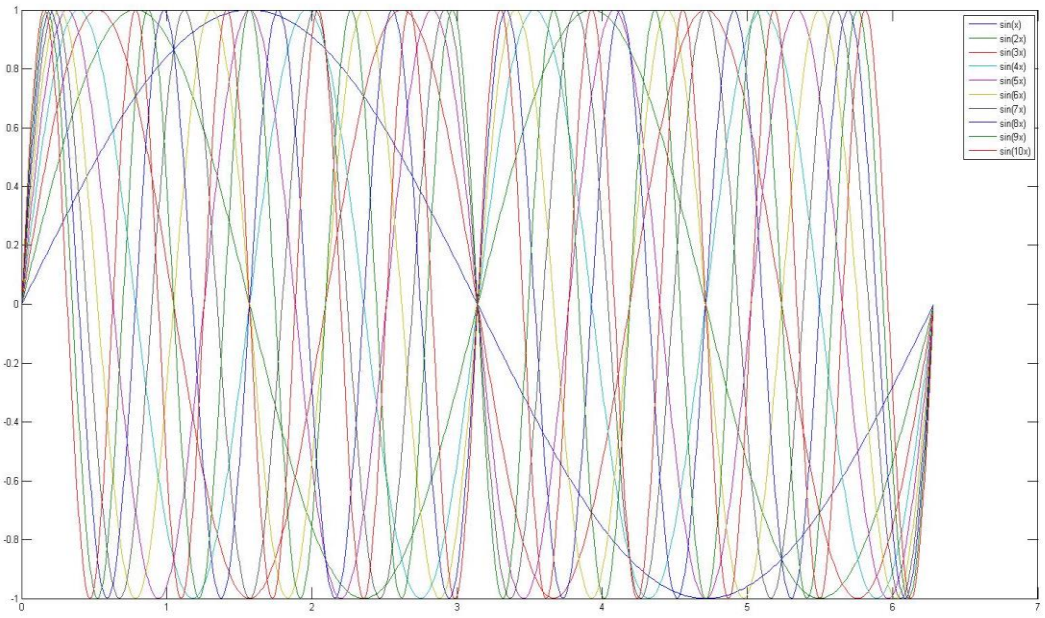
# 河工系 工數二 B 第三次作業解答

1.

請用程式畫 $\left\{ \begin{array}{l} y=1 \\ \begin{array}{ll} y=\cos\theta & y=\cos(2\theta) \\ y=\sin\theta & y=\sin(2\theta) \end{array} \end{array} \right.$ ~ $\begin{array}{l} y=\cos(10\theta) \\ y=\sin(10\theta) \end{array}$ $-2\pi \le \theta \le 2\pi$

(在同一張圖上)



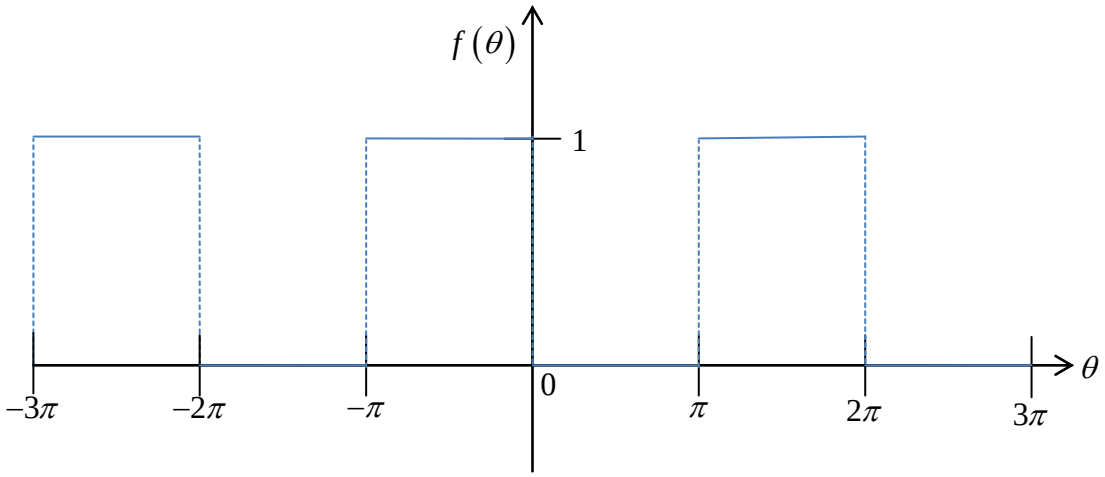



2.

若一周期函數 $f(\theta+2\pi)=f(\theta)$, $f(\theta)=\begin{cases}0, & 0\le\theta\le\pi\\ 1, & \pi\le\theta\le 2\pi\end{cases}$ 如圖，做傅立葉級數可得

$f(\theta)=a_0+\sum_{n=1}^{\infty}a_n\cos(n\theta)+\sum_{n=1}^{\infty}b_n\sin(n\theta)$，請求 $a_0,a_n,b_n$ 並驗證老師是否正確?

請用程式畫有限項 $f(\theta)=\frac{1}{2}+\sum_{n=1}^{N}b_n\sin(n\theta)$ $-2\pi\le\theta\le 2\pi$ 分別當 $N=3,9,11$ 並與下圖做比較



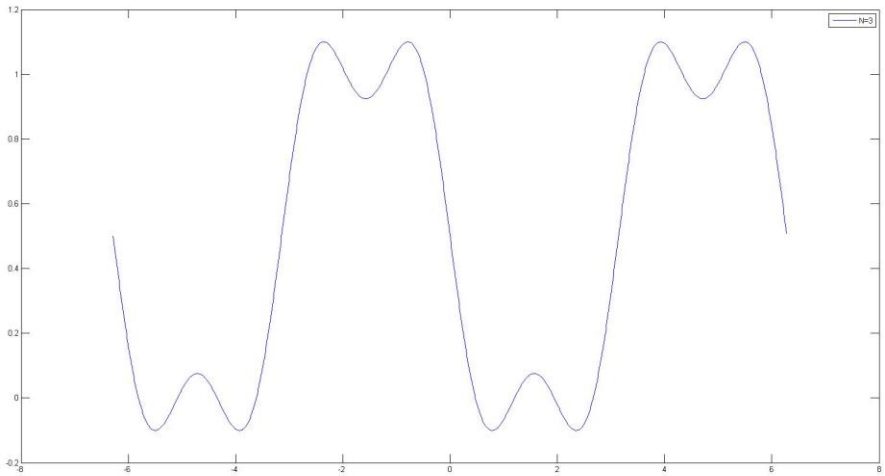

$f(\theta)=\begin{cases}0, & 0\le\theta\le\pi\\ 1, & \pi\le\theta\le 2\pi\end{cases}$

$a_0=\frac{1}{2\pi}\int_{\pi}^{2\pi}f(\theta)d\theta=\frac{1}{2\pi}\int_{\pi}^{2\pi}1d\theta=\frac{\pi}{2\pi}=\frac{1}{2}$

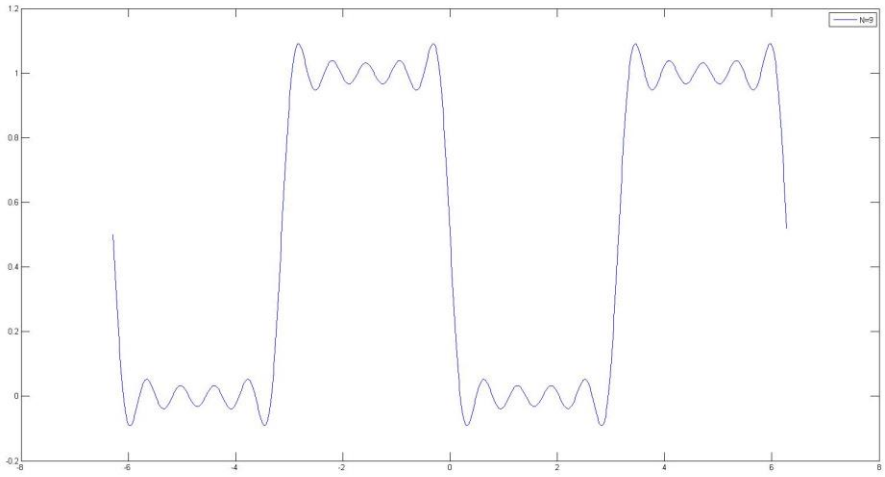
$a_n=\frac{1}{\pi}\int_{\pi}^{2\pi}f(\theta)\cos(n\theta)d\theta=\frac{1}{\pi}\int_{\pi}^{2\pi}\cos(n\theta)d\theta=\frac{1}{n\pi}\sin(n\theta)\Big|_{\pi}^{2\pi}=0$

$b_n=\frac{1}{\pi}\int_{\pi}^{2\pi}f(\theta)\sin(n\theta)d\theta=\frac{1}{\pi}\int_{\pi}^{2\pi}\sin(n\theta)d\theta=-\frac{1}{n\pi}\cos(n\theta)\Big|_{\pi}^{2\pi}=-\frac{1}{n\pi}\left(1-(-1)^n\right)$ ∴正確

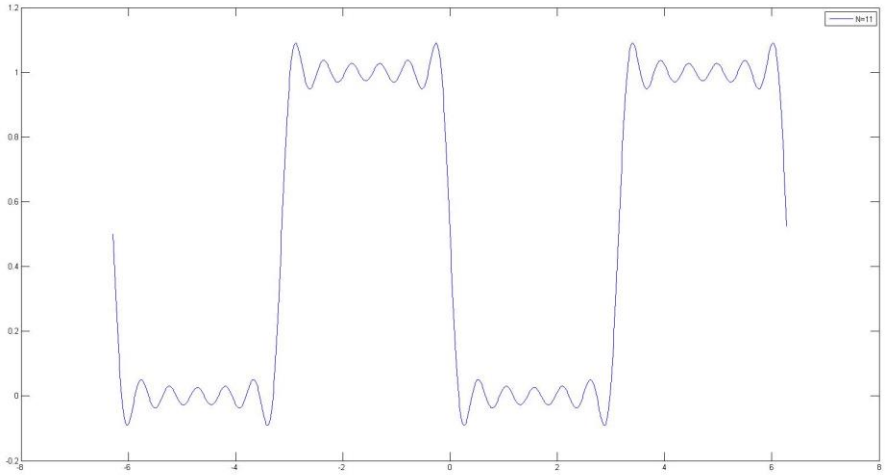
有限項 $N=3$




有限項 $N=9$




有限項 $N=11$




$N$ 越大時越接近

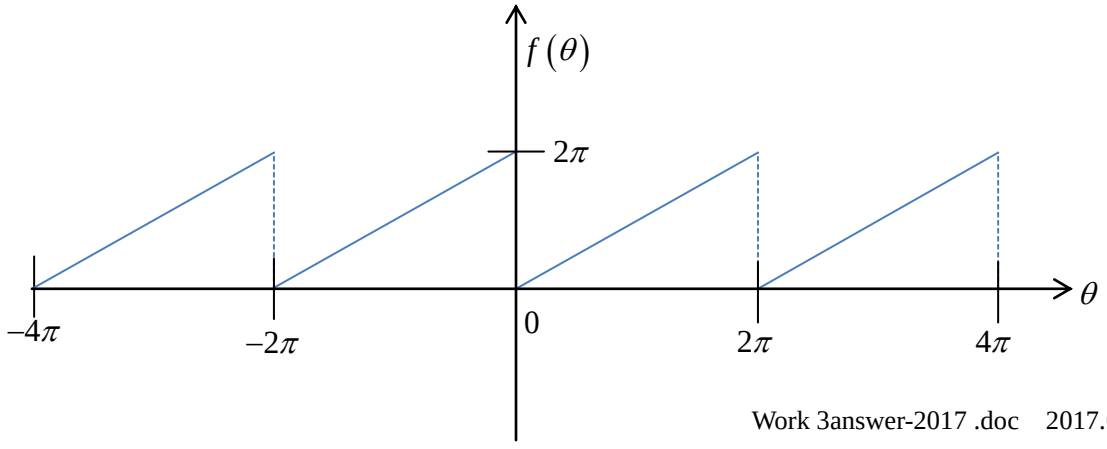
3

(1.)已知 $f(\theta+2\pi)=f(\theta)$ ， $f(\theta)=\theta$ ， $0\le\theta\le2\pi$ ，如圖，做傅立葉級數可得

$f(\theta)=a_0+\sum_{n=1}^{\infty}a_n\cos(n\theta)+\sum_{n=1}^{\infty}b_n\sin(n\theta)$ ，請求 $a_0,a_n,b_n$ ?

(2.) 請用程式畫有限項 $f(\theta)=a_0+\sum_{n=1}^{N}a_n\cos(n\theta)+\sum_{n=1}^{N}b_n\sin(n\theta)$ $-2\pi\le\theta\le2\pi$

分別當$N=3,9,11$ 並與下圖做比較



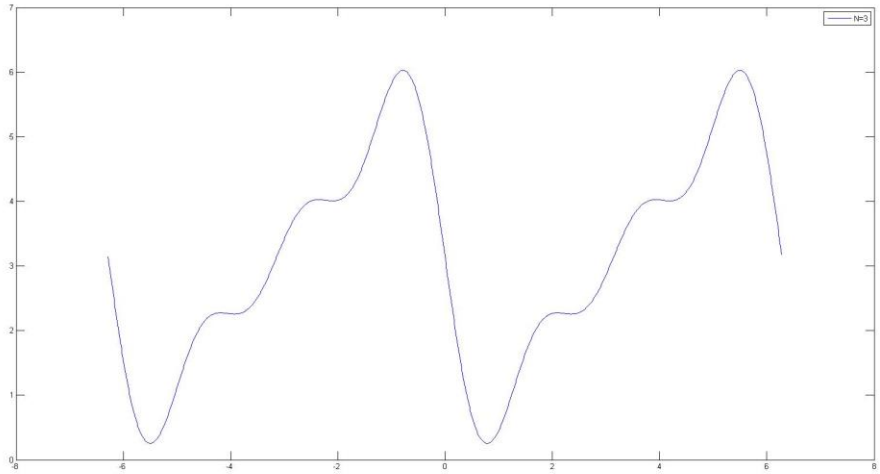
$$f(\theta)=\theta$$

$$a_0=\frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta=\frac{1}{2\pi}\int_0^{2\pi}\theta d\theta=\frac{2\pi^2}{2\pi}=\pi$$

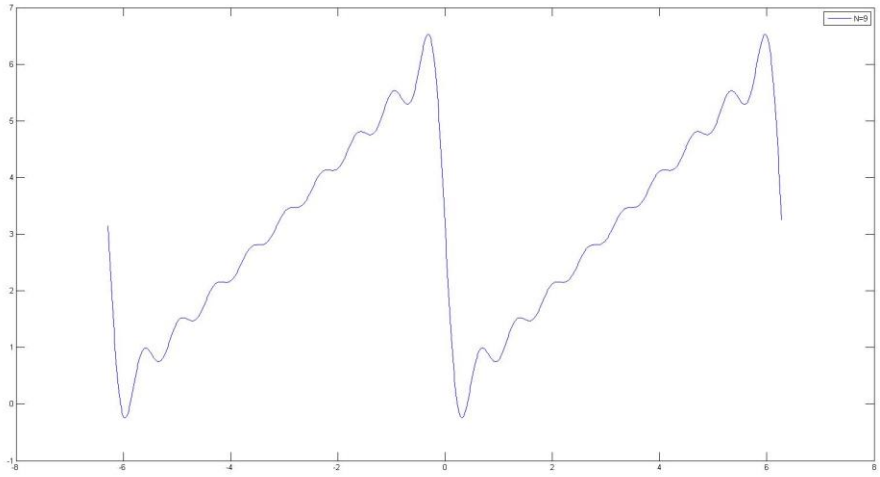
$$a_n=\frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos(n\theta)d\theta=\frac{1}{\pi}\int_0^{2\pi}\theta\cos(n\theta)d\theta=\frac{\theta}{n\pi}\sin(n\theta)\Big|_0^{2\pi}+\frac{1}{n^2\pi}\cos(n\theta)\Big|_0^{2\pi}=0$$

$$b_n=\frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin(n\theta)d\theta=\frac{1}{\pi}\int_0^{2\pi}\theta\sin(n\theta)d\theta=-\frac{\theta}{n\pi}\cos(n\theta)\Big|_0^{2\pi}+\frac{1}{n^2\pi}\sin(n\theta)\Big|_0^{2\pi}=-\frac{2\pi}{n\pi}=-\frac{2}{n}$$

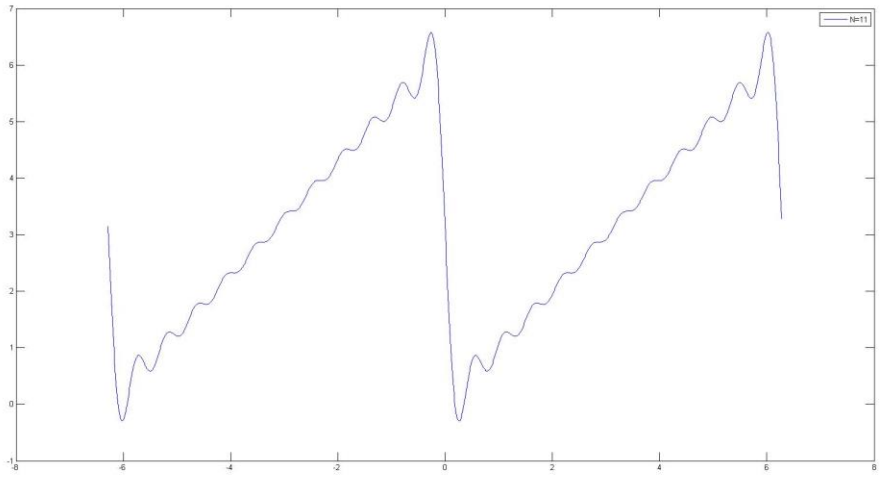
有限項 $N=3$




有限項 $N=9$




有限項 $N=11$


N=11
7
6
5
4
3
2
1
0
-1
-8
-6
-4
-2
0
2
4
6
8