$$f * g = \int_{-\infty}^{\infty} f(u) g(t-u) du \qquad f \circ g = \int_{-\infty}^{\infty} f(u) g(t+u) du$$

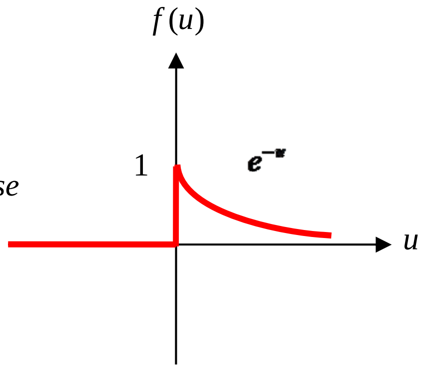
3. Convolution and correlation

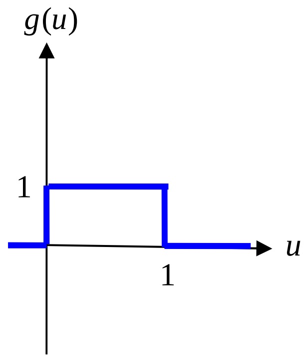
Given $f(u)$ and $g(u)$ function,

$$f(u) = \begin{cases} e^{-u} , u \geq 0 \\ 0 , otherwise \end{cases} \quad \text{and} \quad g(u) = \begin{cases} 1 , 0 \leq u \leq 1 \\ 0 , otherwise \end{cases}$$

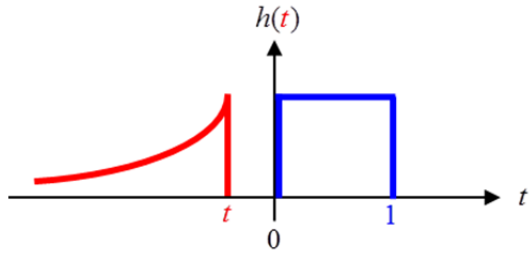
(a) Find (1) $g * f$ (2) $f * g$ (10%)

(b) Find (1) $g \circ f$ (2) $f \circ g$ (10%)



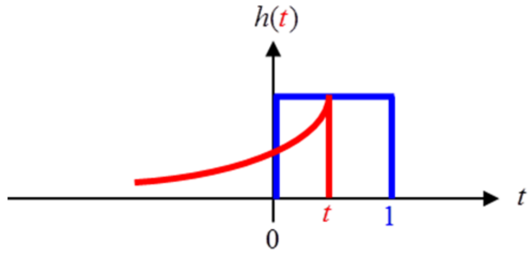


(a) $g * f = \int_{-\infty}^{\infty} g(u) f(t-u) du$



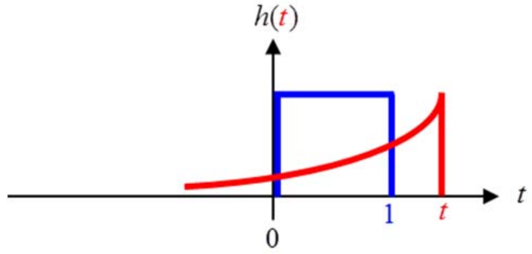
$t < 0$

$$\int_{-\infty}^{\infty} g(u) f(t-u) du = 0$$



$0 \le t \le 1$

$$\int_{-\infty}^{\infty} g(u) f(t-u) du = \int_0^t e^{-(t-u)} du = 1 - e^{-t}$$

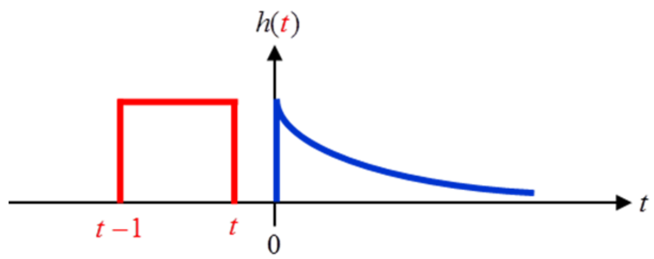


$t > 1$

$$\int_{-\infty}^{\infty} g(u) f(t-u) du = \int_0^1 e^{-(t-u)} du = e^{-t}(e-1)$$

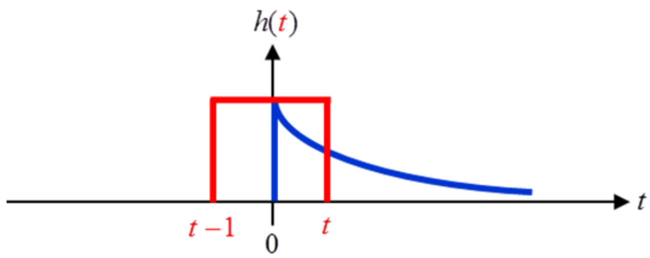
$$\Rightarrow h(t) = g * f = \begin{cases} 0 & , \quad t < 0 \\ 1 - e^{-t} & , \quad 0 \le t \le 1 \\ e^{-t}(e-1) & , \quad t > 1 \end{cases}$$

(b) $f * g = \int_{-\infty}^{\infty} f(u) g(t-u) du$



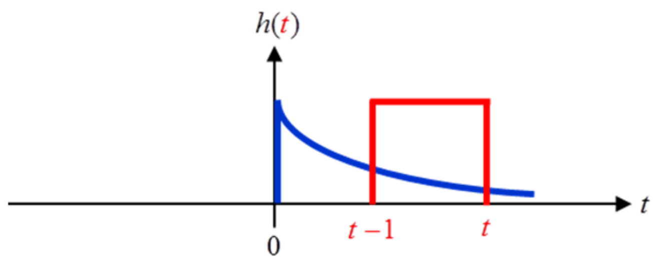
$t < 0$

$$\int_{-\infty}^{\infty} f(u) g(t-u) du = 0$$



$0 \le t \le 1$

$$\int_{-\infty}^{\infty} f(u) g(t-u) du = \int_0^t e^{-u} du = 1 - e^{-t}$$



$t > 1$

$$\int_{-\infty}^{\infty} f(u) g(t-u) du = \int_{t-1}^t e^{-u} du = e^{-t}(e-1)$$

$$\Rightarrow h(t) = f * g = \begin{cases} 0 & , \quad t < 0 \\ 1 - e^{-t} & , \quad 0 \le t \le 1 \\ e^{-t}(e-1) & , \quad t > 1 \end{cases}$$

(c) $g \circ f = \int_{-\infty}^{\infty} f(u)g(t+u)du$

$t < -1$

$$\int_{-\infty}^{\infty} g(u)f(t+u)du = 0$$

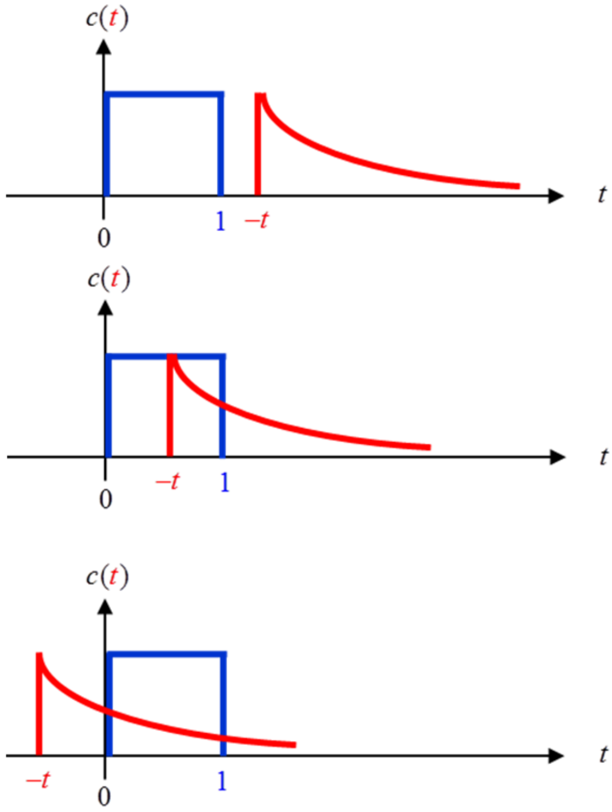
$-1 \le t \le 0$

$$\int_{-\infty}^{\infty} g(u)f(t+u)du = \int_{-t}^{1} e^{-(t+u)}du = 1 - e^{-t-1}$$

$t > 0$

$$\int_{-\infty}^{\infty} g(u)f(t-u)du = \int_{0}^{1} e^{-(t+u)}du = e^{-t}(1-e^{-1})$$

$$\Rightarrow c(t) = g \circ f = \begin{cases} 0 & , \quad t < -1 \\ 1 - e^{-t-1} & , \quad -1 \le t \le 0 \\ e^{-t}(1-e^{-1}) & , \quad t > 0 \end{cases}$$



(d) $f \circ g = \int_{-\infty}^{\infty} f(u)g(t+u)du$

$t < 0$

$$\int_{-\infty}^{\infty} f(u)g(t+u)du = \int_{-t}^{1-t} e^{-u}du = e^{t}(1-e^{-1})$$

$0 \le t \le 1$

$$\int_{-\infty}^{\infty} f(u)g(t+u)du = \int_{0}^{1-t} e^{-u}du = 1 - e^{t-1}$$

$t > 1$

$$\int_{-\infty}^{\infty} f(u)g(t+u)du = 0$$

$$\Rightarrow c(t) = f \circ g = \begin{cases} e^{t}(1-e^{-1}) & , \quad t < 0 \\ 1 - e^{t-1} & , \quad 0 \le t \le 1 \\ 0 & , \quad t > 1 \end{cases}$$