Sol. 6

$$L[\dot{y}(t)+y(t)]=L(0)$$
$$\Rightarrow sY(s)-y(0)+Y(s)=0$$
$$\Rightarrow (s+1)Y(s)=y(0)=1$$
$$\Rightarrow Y(s)=\frac{1}{s+1}$$
$$\Rightarrow y(t)=L^{-1}\left[\frac{1}{s+1}\right]=e^{-t}$$

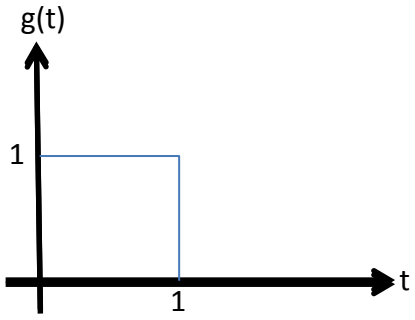
Laplace transform 求解 ODE

Sol. 7

(1)

$$\begin{cases}\dot{y}(t)+y=g(t)\\ y(0)=0\end{cases}$$

$$g(t)\begin{cases}1, 0\le t\le 1\\ 0, otherelse\end{cases}$$



方法(一) (工數一)

1) $0<t<1$

$$\begin{cases}\dot{y}(t)+y(t)=1\\ y(0)=0\end{cases}$$

$$\dot{y}_c(t)+y_c(t)=0$$

$$\dot{y}_c(t)=-y_c(t)$$

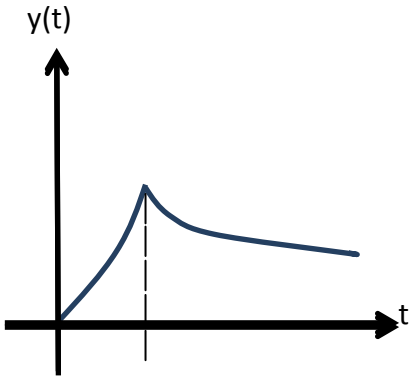
$$y_c(t)=c\cdot e^{-t}$$
$$\dot{y}_p(t)+y_p(t)=1$$
$$y_p(t)=1$$

$y_{total}(t) = c \cdot e^{-t} + 1$

$y(0) = c + 1 = 0$

$c = -1$

$y_{total}(t) = -e^{-t} + 1$



2) $t > 1$

$$\begin{cases} \dot{y}(t) + y(t) = 0 \\ y(1) = -e^{-1} + 1 \end{cases}$$

$y(t) = c \cdot e^{-t}$

$y(1) = c \cdot e^{-1} = -e^{-1} + 1$

$c = \dfrac{-e^{-1} + 1}{e^{-1}} = e - 1$

$y(t) = (e-1)e^{-t}$

驗算：

(1)
$$\dot{y}(t) + y(t) = (e^{-t}) + (-e^{-t} + 1) = 1$$
$$y(0) = -1 + 1 = 0$$

(2)
$$\dot{y}(t) + y(t) = \frac{e^{-1} - 1}{e^{-1}} e^{-t} + \frac{-e^{-1} + 1}{e^{-1}} e^{-1} = 0$$
$$y(1) = \frac{-e^{-1} + 1}{e^{-1}} e^{-1} = -e^{-1} + 1$$

(2)

$$G_1(s) = \frac{1}{s},\ G_2(s) = \frac{e^{-s}}{s},\ G(s) = \frac{1 - e^{-s}}{s}$$

(3)

方法(二)

$L[\dot{y}(t) + y(t)] = L[g(t)]$

$$sY(s) - y(0) + Y(s) = G(s) = \int_0^{\infty} g(t) \cdot e^{-st} dt = \int_0^1 1 \cdot e^{-st} dt + \int_1^{\infty} 0 \cdot e^{-st} dt = \frac{1 - e^{-s}}{s}$$

$$\Rightarrow Y(s)=\frac{1-e^{-s}}{s(s+1)}=\frac{1}{s(s+1)}-\frac{e^{-s}}{s(s+1)}$$

$$=(\frac{1}{s}-\frac{1}{s+1})-(\frac{e^{-s}}{s}-\frac{e^{-s}}{s+1})$$

$$0<t<1\Rightarrow y(t)=(1-e^{-t})-(0-0)=1-e^{-t}$$

$$t>1\Rightarrow y(t)=(1-e^{-t})-(1-e^{-(t-1)})=-e^{-t}+e\cdot e^{-t}=(e-1)e^{-t}$$

方法(三)

$$Y(s)=F(s)G(s)$$

$$y(t)=f(t)*g(t)$$

$$sY(s)-y(0)+Y(s)=G(s)\Rightarrow$$

$$Y(s)=\frac{G(s)}{s+1}=F(s)G(s),(F(s):\frac{1}{s+1})$$

$$f(t)=e^{-t}$$

$$g(t)=\begin{cases}1,0\le t\le 1\\0,otherelse\end{cases}$$

$$f*g=\begin{cases}0,t<0\\1-e^{-t},0<t<1\\(e-1)e^{-t},1<t<\infty\end{cases}$$