二階與高階微分方程式

海大河海系 陳正宗

First order ODE

$$y' + a(x)y = f(x), y(x_0) = y_0$$

Second order ODE

$$y'' + a(x)y' + b(x)y = f(x), y(x_0) = y_0, y'(x_0) = y_1$$

nth order ODE

$$y^n(x) + a_{n-1}(x)y^{n-1}(x) + \cdots + a_1(x)y'(x) + a_0(x)y(x) = f(x)$$

initial conditions:

$$y(x_0) = y_0, y'(x_0) = y_1, \cdots, y^{n-1}(x_0) = y_{n-1}$$

Homogeneous if $f(x) = 0$, otherwise nonhomogeneous.

Linearity if y_1 and y_2 both satisfy the homogeneous ODE, then $y_1 + y_2$ satisfies the homogeneous ODE.

Existence and uniqueness theorem: if $a_0(x), \cdots a_{n-1}(x), f(x)$ are all continuous on the interval (x_1, x_2).

No. of independent solutions

$$y'' - y = 0$$

Sol.: $y(x) = e^x, e^{-x}, cosh(x), sinh(x)$ all satisfy the ODE

only two conditions to determine the coefficients.

what is wrong ?

Linear independence and dependence

$c_1y_1(x) + c_2y_2(x) = 0 \rightarrow$ only choice of $c_1 = c_2 = 0$.

$c_1y_1(x) + c_2y_2(x) = 0 \rightarrow c_1 \neq 0$ or $c_2 \neq 0$.

Vector space : $(0, 1)$ and $(1, 0)$

Function space : e^x, e^{-x}, $cosh(x)$ and $sinh(x)$.