Two sources: external excitation or free vibration with two near frequencies

External excitation

$$\ddot{x}(t)+\omega^2 x(t) = F cos(\Omega t)$$

General solution

$$x(t) = c_1 cos(\omega t)+c_2 sin(\omega t)+\frac{F}{(\omega^2-\Omega^2)} cos(\Omega t)$$

Initial conditions to determine c_1 and c_2

$$x(0) = 0, \dot{x}(0) = 0$$

Total solution

$$x(t) = \frac{F}{(\omega^2-\Omega^2)}\{cos(\Omega t)-cos(\omega t)\}$$

Stage 1: excitation

$$x(t) = \frac{F}{(\omega^2-\Omega^2)}\{cos(\Omega t)-cos(\omega t)\}$$

Stage 2: beating ($\omega \doteq \Omega$)

$$x(t) = \frac{F}{(\omega^2-\Omega^2)}\{cos(\Omega t)-cos(\omega t)\}$$

$$x(t) = \frac{-2F}{(\omega^2-\Omega^2)} sin(\frac{\Omega+\omega}{2})t\, sin(\frac{\Omega-\omega}{2})t \rightarrow beating$$

$$x(t) = \frac{2F}{(2\omega)(2\epsilon)} sin(\omega t)\, sin(\epsilon t)$$

$$x(t) = \frac{Ft}{2\omega} sin(\omega t) \rightarrow resonance$$

where $\Omega-\omega = 2\epsilon$.

Stage 3: resonance ($\omega = \Omega$)

$$x(t) = \frac{F}{(\omega^2-\Omega^2)}\{cos(\Omega t)-cos(\omega t)\} \rightarrow \infty$$

$$x(t) = \frac{Ft}{2\omega} sin(\omega t) \rightarrow \infty, as\ t \rightarrow \infty$$