Frame indifference: objectivity

Problem: a SDOF mass-spring system with a mass m and a spring constant k in equilibrium. A half mass drops out suddenly, Please describe the vibration phenomenon.

Reference 1: $x_1(t)$ positive downward from original length of spring

$$governing\ equation\ \frac{m}{2}\ddot{x}_1(t)+kx_1(t)=\frac{mg}{2}$$

$$initial\ conditions\ x_1(0)=mg/k, \dot{x}_1(0)=0$$

$$solution\ x_1(t)=\frac{mg}{2k}cos(\omega_1 t)+\frac{mg}{2k}>0, for\ any\ t$$

where $\omega_1^2=2k/m$.

Reference 2: $x_2(t)$ positive downward from equilibrium point

$$governing\ equation\ \frac{m}{2}\ddot{x}_2(t)+k(x_2(t)+mg/k)=\frac{mg}{2}$$

$$initial\ conditions\ x_2(0)=mg/k, \dot{x}_2(0)=0$$

$$solution\ x_2(t)=\frac{mg}{2k}cos(\omega_1 t)-\frac{mg}{2k}$$

Reference 3: $x_3(t)$ positive downward from the lowerest point

$$governing\ equation\ \frac{m}{2}\ddot{x}_3(t)+k(x_2(t)+2mg/k)=\frac{mg}{2}$$

$$initial\ conditions\ x_3(0)=-mg/k, \dot{x}_3(0)=0$$

$$solution\ x_3(t)=\frac{mg}{2k}cos(\omega_1 t)-\frac{3mg}{2k}$$

Discussions:

$$x_1(t)=x_2(t)+\frac{mg}{k}=x_3(t)+\frac{2mg}{k}$$

All the three solutions are the same in the physical sense.

All the three solutions obeys the objectivity, i.e. frame indifference

Please plot the three solutions by mathematica.