Two sources: external excitation or free vibration with two near frequencies

External excitation

$$\ddot{x}(t)+\omega^2 x(t) = Fcos(\omega t)$$

By variation of parameters:

$$x(t) = u_1 cos(\omega t)+u_2 sin(\omega t)$$

Two constraints:

$$cos(\omega t)u_1' + sin(\omega t)u_2' = 0$$

$$-\omega sin(\omega t)u_1' + \omega cos(\omega t)u_2' = Fcos(\omega t)$$

Solve u_1', u_2'

$$u_1'(t) = \frac{-F}{2\omega} sin(2\omega t)$$

$$u_2'(t) = \frac{F}{2\omega}(1+cos(2\omega t))$$

Solve u_1, u_2

$$u_1(t) = \frac{F}{4\omega^2} cos(2\omega t)+c_1$$

$$u_2(t) = \frac{F}{2\omega}(t+\frac{1}{2\omega} sin(2\omega t))+c_2$$

Particular solution contains a complementary solution:

$$x(t) = \frac{Ft}{2\omega} sin(\omega t)+\frac{F}{4\omega^2} cos(\omega t)$$

Particular solution:

$$x(t) = \frac{Ft}{2\omega} sin(\omega t) \rightarrow resonance$$