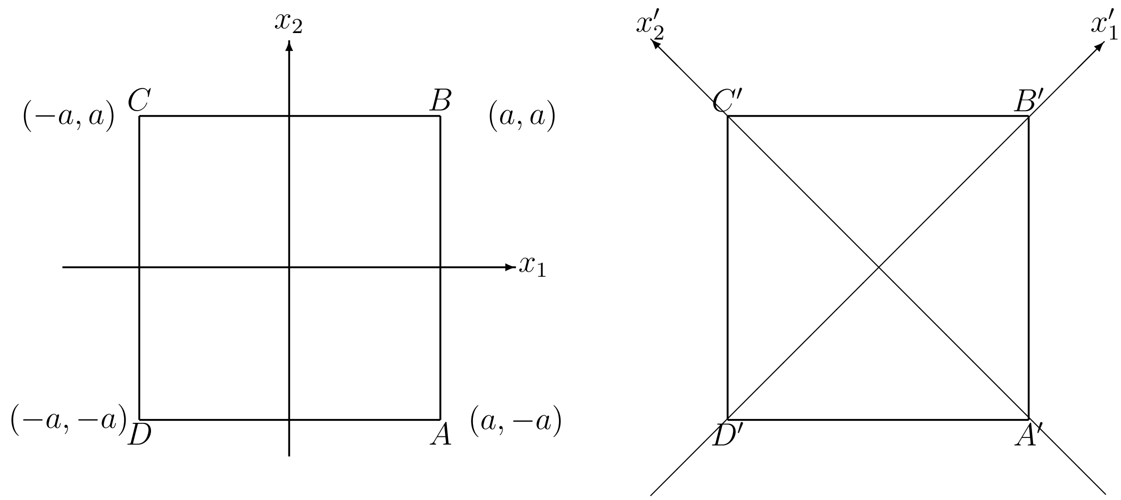
# Scalar, Vector and Tensor




$x_1$-$x_2$ coordinate system $x_1'$-$x_2'$ coordinate system

1. Square $ABCD$ can be described in $x_1x_2$ and $x_1'x_2'$ coordinates.

   (1). Express the coordinates for $A'$, $B'$, $C'$ and $D'$ in terms of $x_1', x_2'$ coordinate system.

   (2). Determine the length of $\overline{AC}$ and $\overline{BD}$ in both coordinates. Discuss the result.

   (3). Determine the vectors of $\overrightarrow{AC}$ and $\overrightarrow{BD}$ in both coordinates. Also, find the inner and outer products of $\overrightarrow{AC}$ and $\overrightarrow{BD}$. Any difference between the two coordinate discriptions? Find the area of $ABCD$.

   (4). If a vector can be expressed by $(v_1, v_2)$ and $(v_1', v_2')$ in $x_1 - x_2$ and $x_1' - x_2'$ coordinate system, respectively, find the matrix $T$ if

   $$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = T \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

   (5). Find $I_{11}$, $I_{12}$, $I_{21}$ and $I_{22}$ if $I_{ij} = \int_A x_i x_j dA$.

   (6). Find $I_{1'1'}$, $I_{1'2'}$, $I_{2'1'}$ and $I_{2'2'}$ if $I_{i'j'} = \int_A x_{i'} x_{j'} dA$.

   (7). If moment of inertia can be expressed by $I_{11}, I_{12}, I_{21}, I_{22}$ and $I_{1'1'}, I_{1'2'}, I_{2'1'}, I_{2'2'}$ in $x_1 - x_2$ and $x_1' - x_2'$ coordinate system, respectively, find the matrix $A$ if

   $$\begin{bmatrix} I_{1'1'} & I_{1'2'} \\ I_{2'1'} & I_{2'2'} \end{bmatrix} = A^T \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix} A$$

   (8). Compare $I_{11} + I_{22}$ with $I_{1'1'} + I_{2'2'}$

   (9). Compare

   $$det \begin{vmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{vmatrix} \quad with \quad det \begin{vmatrix} I_{1'1'} & I_{1'2'} \\ I_{2'1'} & I_{2'2'} \end{vmatrix}$$

   (10). Find the eigenvalues and eigenvectors for the two matrix.

2. Classify all the calulated quantities to scalar, vector or rank-2 tensor.

| Tensor rank | name | physical quantity |
|---|---|---|
| 0 | scalar | |
| 1 | vector | |
| 2 | tensor(2) | |