Digital Simulation of the Transformation of Plane Stress

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ABSTRACT: In this study, we developed a computer program to simulate the transformation of plane stress by using Visual Basic.NET. We applied the equations of stress transformation to plane stress problems to calculate the stresses with respect to the $1-2$ axes, which are rotated counterclockwise through an angle θ about the $x-y$ origin, and showed the visual results on the screen. In addition, we used animation to observe the change of plane stress. This program was then used in teaching courses, such as Mechanics of Materials and Linear Algebra. Use of the software may help students to understand principal stresses, principal axes, Mohr's circle, eigenvalues, eigenvectors, similar matrices, and invariants. ©2008 Wiley Periodicals, Inc. Comput Appl Eng Educ 17: 25–33, 2009; Published online in Wiley InterScience (www.interscience.wiley.com); DOI 10.1002/cae.20180

Keywords: plane stress; principal stresses; principal axes; Mohr's circle; eigenvalues; eigenvectors; similar matrices; invariants

INTRODUCTION

Computer simulations are methods to help students understand a new concept quickly. They have been proven to be an efficient tool both in teaching and self learning [1]. Many researchers have developed computer software for educational purposes. Vidaurre et al. [2] developed simulation programs for curve fitting, wheel motion, and frictions. Lee [3] developed a series of physics simulations. Chimenti and Ochs [4] developed a 3-D simulator for moments of inertia.

This article reports on a digital simulation for the transformation of plane stress. We chose this topic because this simulation can be used to teach many concepts that are not straightforward for students to comprehend.

In linear algebra, students who learn eigenvalue problems for the first time usually have difficulties in understanding the problems. However, eigenvalue problems are very important in science and engineering. So, we designed a simulation to illustrate the eigenvalue problem through plane stress

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transformation. For each different problem, eigenvalues usually have special physical meanings. Moreover, the corresponding eigenvectors usually indicate directions that have special meanings. If a graphical demonstration is given, students will then have a better understanding of the meaning of eigenvalues and eigenvectors.

Teachers generally agree that some concepts in eigenvalue problems often confuse students. Therefore, we try to provide a tool for students to observe the meaning of the problems. With Visual Basic.NET [5] as our platform, we use transformation of plane stress problems to illustrate eigenvalue problems. The stress status in an infinitesimal element of a continuum material can be described as a symmetric matrix [6]. Through graphical illustration, students can easily understand some difficult concepts of eigenvalue problems, such as: similar matrices, physical meaning of eigenvalues, physical meaning of eigenvectors, invariance of trace, invariance of determinant, principal stress, principal direction, maximum shear stress, average stress, etc.

THEORY IN TRANFORMATION OF PLANE STRESS

The theory of transformation of plane stress has been developed for a long time and is well established [6]. We will briefly state it for convenience.

A general three-dimensional state of stress of an infinitesimal element in a continuum material can be expressed by nine stress components σ_{ij} (where i, $j = x$, y, or z), as shown in Figure 1. According to conventional subscript notation, when $i = j$, the stress component is a normal stress; and when $i \neq j$, the stress component is a shear stress. The first subscript represents the direction of the outward normal to the face on which the stress component acts, and the second subscript refers to the direction in which the stress component itself acts [7].

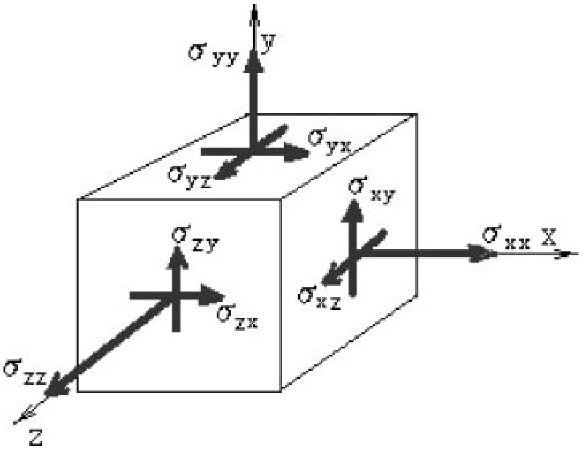


Figure 1 General three-dimensional stresses acting on an infinitesimal element of a continuum material with respect to the $x-y-z$ axes.

It is to be noted that σ_{ij} is equal to σ_{ji} because of the reciprocity of shear stress. This means that if we write σ_{ij} in matrix form, it will be a symmetric matrix. It is also to be noted that σ_{ii} is a tension stress when its value is positive and σ_{ii} is a compressive stress when its value is negative. On the other hand, σ_{ij} is a shear component no mater whether its value is positive or negative.

In our study, we will concentrate on plane stress problems instead of general three-dimensional problems because two-dimensional plane stress problems are easier to explain to students and easier to illustrate on a computer screen.

In order to describe the stress state of an infinitesimal element of a continuum material under plane stresses, we need a 2 by 2 stress matrix:

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \tag{1}$$

The four entries in the stress matrix (1) represent the stresses acting on the element with respect to the $x-y$ coordinates and are shown in Figure 2. The values of σ_{xy} and σ_{yx} are the same, due to reciprocity of shear stresses in Mechanics of Materials. Hence, the stress matrix (1) is symmetric. That is, there are only three independent entries in the stress matrix. The normal stresses acting on the opposite side of the element are equal because the element is vanishingly small.

If we describe the stress state of the element using a new $1-2$ coordinate system, which is rotated

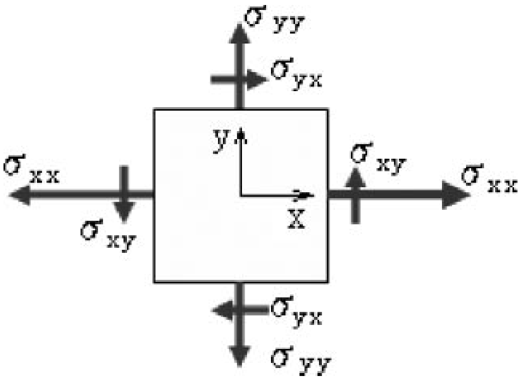


Figure 2 The plane stresses acting on an infinitesimal element of a continuum material with respect to the $x-y$ axes.

counterclockwise through an angle θ about the $x-y$ origin, we need another 2 by 2 symmetric matrix:

$$\sigma' = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \tag{2}$$

The four entries in the stress matrix (2) represent the stresses acting on the element with respect to the 1−2 axes and are shown in Figure 3. Similarly, there are only three independent entries in the stress matrix, since the matrix is symmetric. The 1−2 axes are rotated counterclockwise through an angle θ about the $x-y$ origin.

The two stress matrices mentioned above actually describe the same physical phenomenon with respect to different coordinate systems. Therefore, they are related to each other. We will now derive the relation between them.

Consider the vanishingly small element shown in Figure 4. The inclined surface has an outward normal pointing to the 1 axis, which is rotated from the x axis counterclockwise through an angle θ. From the force equilibrium equations in the horizontal direction and vertical direction, we can obtain the following equations

$$\begin{aligned}\sigma_{11}\frac{A}{\cos\theta}\cos\theta - \sigma_{12}\frac{A}{\cos\theta}\sin\theta \\ - \sigma_{xx}A - \sigma_{xy}A\frac{\sin\theta}{\cos\theta} = 0\end{aligned} \tag{3}$$

$$\begin{aligned}\sigma_{11}\frac{A}{\cos\theta}\sin\theta + \sigma_{12}\frac{A}{\cos\theta}\cos\theta \\ - \sigma_{xy}A - \sigma_{yy}A\frac{\sin\theta}{\cos\theta} = 0,\end{aligned} \tag{4}$$

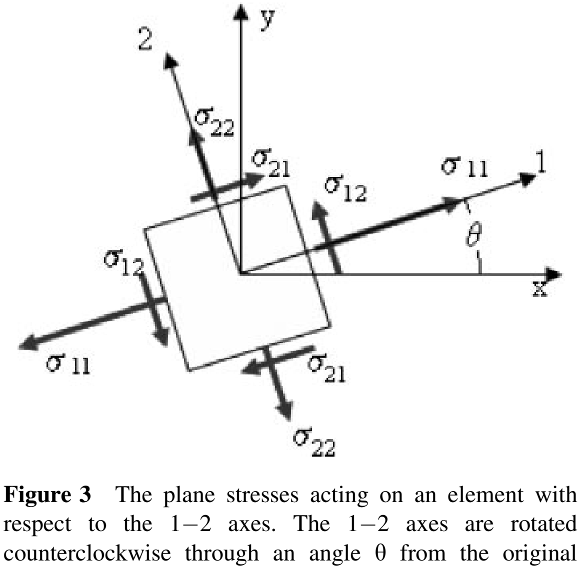


Figure 3 The plane stresses acting on an element with respect to the 1−2 axes. The 1−2 axes are rotated counterclockwise through an angle θ from the original $x-y$ axes.

where A is the area of the vertical surface shown in Figure 4 [8].

Simplifying the above two equations, we have

$$\sigma_{11}\cos\theta - \sigma_{12}\sin\theta = \sigma_{xx}\cos\theta + \sigma_{xy}\sin\theta \tag{5}$$

$$\sigma_{11}\sin\theta + \sigma_{12}\cos\theta = \sigma_{xy}\cos\theta + \sigma_{yy}\sin\theta \tag{6}$$

Solving the system of linear Equations (5) and (6), we have

$$\sigma_{11} = \sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta \tag{7}$$

$$\sigma_{12} = (\sigma_{yy} - \sigma_{xx})\sin\theta\cos\theta - \sigma_{xy}(\sin^2\theta - \cos^2\theta), \tag{8}$$

The value of σ_{22} can be derived from Equation (7) by changing θ to θ + 90°, since the 2 axis is 90° counterclockwise from the 1 axis. Thus, we have

$$\sigma_{22} = \sigma_{xx}\sin^2\theta - 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\cos^2\theta, \tag{9}$$

Equations (7−9) state the relation between the two stress matrices and are widely used in Mechanics of Materials [6−9].

As can be seen from Equation (7), the normal stress is a function of θ. That is, the normal stress varies with angle θ. The maximum and minimum values of the normal stress are called the principal stresses. If we differentiate σ_{11} in Equation (7) with respect to θ and set it equal to zero, we obtain

$$\frac{d\sigma_{11}}{d\theta} = -(\sigma_{xx} - \sigma_{yy})\sin 2\theta + 2\sigma_{xy}\cos 2\theta = 0. \tag{10}$$

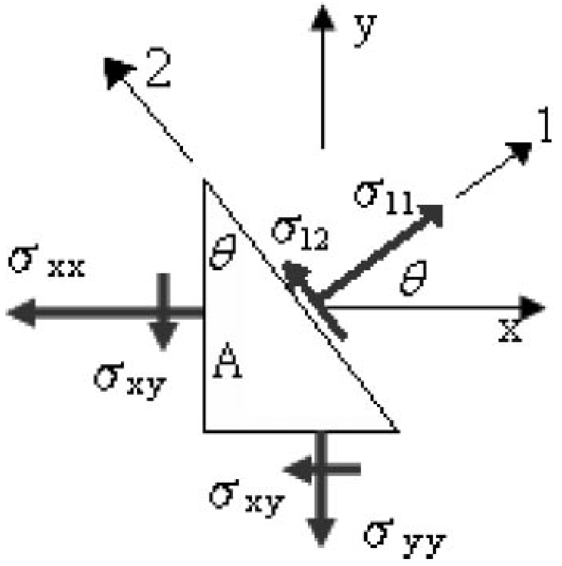


Figure 4 The stresses acting on an element with an inclined side whose normal has an angle +θ from the x axis.

Solving the above equation, we have

$$\tan 2\theta_{\mathrm{p}} = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}}, \tag{11}$$

in which θ_{p} is used in place of θ to denote the angles defining the principle axes.

From Equation (11), two values of θ_{p} can be obtained. The two θ_{p} differ with an angle 90° from the definition of tangent. For one of the two values of θ_{p}, the normal stress is a maximum, and for the other the normal stress is a minimum. That is, when σ_{11} reaches its maximum, σ_{22} will reach a minimum because the two θ_{p} differ with an angle 90°. Similarly, if σ_{11} is a minimum, σ_{22} will be a maximum. Substituting θ_{p} into Equation (8), we can also find that when the normal stresses reach maximum or minimum, the shear stress will be zero.

Rearranging Equations (7) and (8), we can obtain

$$\left(\sigma_{11} - \frac{\sigma_{xx} + \sigma_{yy}}{2}\right)^2 + \sigma_{12}^2 = \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2. \tag{12}$$

The above equation is a circle of radius $\sqrt{((\sigma_{xx} - \sigma_{yy})/2)^2 + \sigma_{xy}^2}$ with a center at $((\sigma_{xx} + \sigma_{yy})/2, 0)$. It was developed by Mohr and is called Mohr's circle [6]. Mohr's circle provides an alternative way to solve stress transformation problems. This graphical illustration, although developed more than 100 years ago, is still widely used in engineering. Thus, we also provide this circle in the program for user reference.

Summarizing Equations (7–9) in linear algebraic form, we have the following equation:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \tag{13}$$

or in simplified form

$$\sigma' = T^{-1}\sigma T$$

Note that the first matrix in the right hand side of Equation (13) is an inverse of a transformation matrix T. In some articles [10], the inverse is replaced by a transpose as follows:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{t} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{14}$$

Equations (13) and (14) are both correct, because the transformation matrix T in these two equations is an orthogonal matrix. That is, the two columns in the transformation matrix T are orthogonal to each other and have unit length. In linear algebra, the inverse of an orthogonal matrix equals its transpose.

We can also find two invariants I_1 and I_2 from Equations (7–9).

$$I_1 = \sigma_{11} + \sigma_{22} = \sigma_{xx} + \sigma_{yy} = \mathrm{trace}(\sigma) = \mathrm{trace}(\sigma') \tag{15}$$

$$\begin{aligned} I_2 = \sigma_{11}\sigma_{22} - \sigma_{12}^2 = \sigma_{xx}\sigma_{yy} - \sigma_{xy}^2 \\ = \det(\sigma) = \det(\sigma') \end{aligned} \tag{16}$$

From Equation (15), we note that when σ_{11} is a maximum, σ_{22} will be a minimum, since the sum of σ_{11} and σ_{22} is an invariant.

In linear algebra, a general eigenvalue problem can be stated as

$$A\underline{x} = \lambda\underline{x}, \tag{17}$$

where A is an n by n matrix, λ is the eigenvalue of A, and the vector $\underline{x}$ is the corresponding eigenvector of λ. Note that if $\underline{x}$ is a zero vector, Equation (17) is automatically satisfied. However, if $\underline{x}$ is a zero vector, it is not an eigenvector by definition [11].

In our case, the matrix A is replaced by the 2 by 2 stress matrix with respect to the x–y axes. The explicit form of the eigenvalue problem becomes

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{18}$$

Moving the right hand side to the left, we have

$$\begin{bmatrix} \sigma_{xx} - \lambda & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{19}$$

In order to get a nonzero solution of x_1 and x_2 for the above system of linear equations, the determinant of the matrix in Equation (19) must be zero. Thus, we have

$$\lambda^2 - (\sigma_{xx} + \sigma_{yy})\lambda + (\sigma_{xx}\sigma_{yy} - \sigma_{xy}^2) = 0 \tag{20}$$

Equation (20) is called the characteristic equation. The roots of the equation are the eigenvalues. Because the characteristic equation of the two-dimensional stress matrix is second order, we have two eigenvalues, namely λ_1 and λ_2. Substituting the two eigenvalues into Equation (19), we can solve the corresponding two eigenvectors: $\underline{e_1}$ and $\underline{e_2}$. Thus, we have

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \underline{e_1} = \lambda_1 \underline{e_1} \tag{21}$$

$$\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}\end{bmatrix}\underline{e_2} = \lambda_2\underline{e_2} \tag{22}$$

Rearranging Equations (21) and (22), we obtain

$$\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix} = \begin{bmatrix}\underline{e_1} & \underline{e_2}\end{bmatrix}^{-1}\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}\end{bmatrix}\begin{bmatrix}\underline{e_1} & \underline{e_2}\end{bmatrix} \tag{23}$$

Comparing Equations (13) and (23), we note that if we choose an appropriate angle θ, the transformation matrix in Equation (13) will consist of the eigenvectors in its columns, and the left hand side of Equation (13) will be diagonalized.

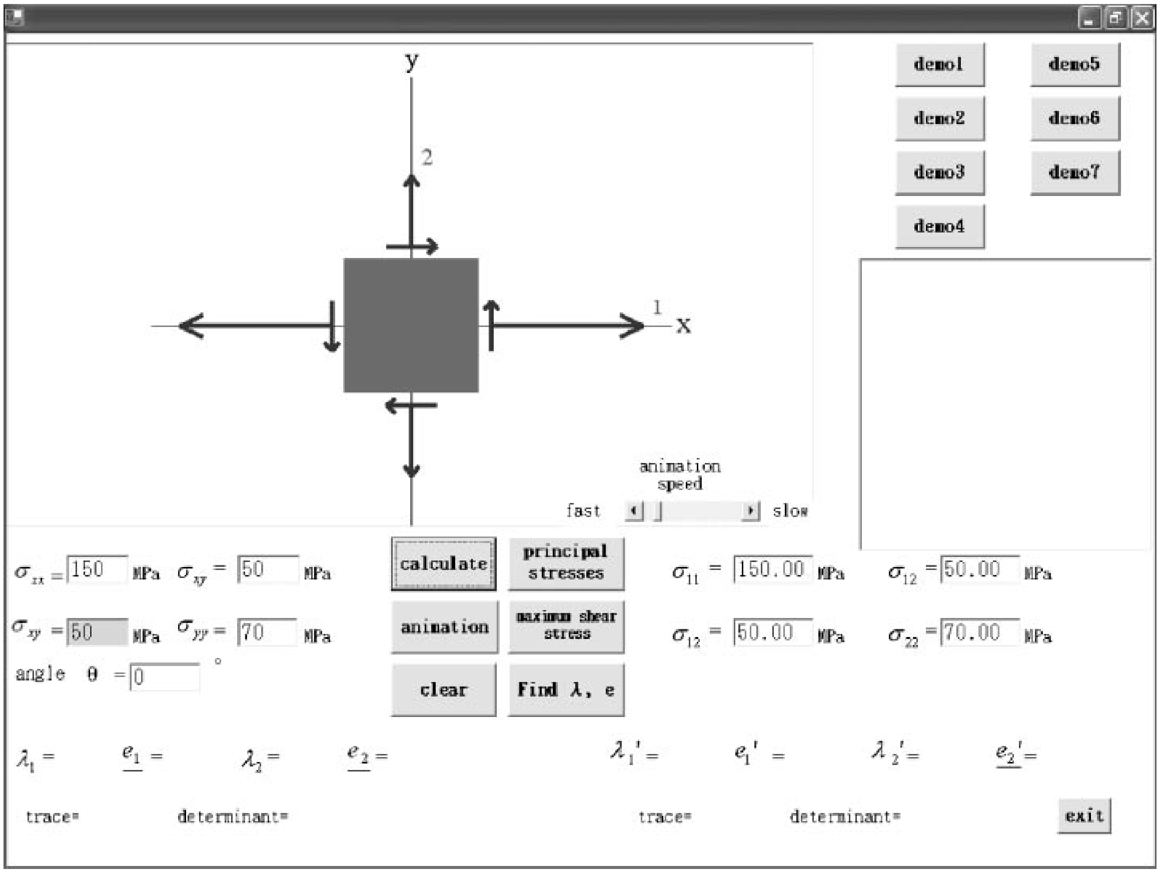
Since the two stress matrices describe the same phenomenon, they share many common properties. In linear algebra, the two matrices are called similar matrices. They have the same eigenvalues, the same trace, the same determinant, but different eigenvectors. Although their eigenvectors are different in mathematical form, the directions of the eigenvectors are actually the same in real space.

It is also to be noted from linear algebra that the two similar matrices of size 2 by 2 have two invariants, namely: trace and determinant [11]. The two invariants can be derived from Equation (20). Their mathematical expressions are stated in Equations (15) and (16).

For three dimensional stress problems, we will still obtain an equation similar to Equation (13). However, we will have three eigenvalues and three eigenvectors. We will also have three invariants for three dimensional problems.

PROGRAM IMPLEMENTATION

A digital simulation program was then written from the above theory by using Microsoft Visual.Basic.-NET. Figure 5 shows the appearance of the program on the screen. The stress matrix with respect to $x-y$ axes can be entered from the lower left corner. Also, the angle of $1-2$ axes with respect to the original $x-y$ axes can also be entered from the lower left corner. We can now click the "calculate" button to calculate the stress matrix with respect to $1-2$ coordinates. Then the computer will calculate the new stress matrix



σ_{xx} = 150 MPa σ_{xy} = 50 MPa

σ_{xy} = 50 MPa σ_{yy} = 70 MPa

angle θ = 0 °

σ_{11} = 150.00 MPa σ_{12} = 50.00 MPa

σ_{12} = 50.00 MPa σ_{22} = 70.00 MPa

λ_1 = $\underline{e_1}$ = λ_2 = $\underline{e_2}$ = λ_1' = e_1' = λ_2' = $\underline{e_2}'$ =

trace= determinant= trace= determinant=

Figure 5 The appearance of the program on the screen. The stress matrix with respect to the $x-y$ axes can be entered from the lower left corner. The angle of $1-2$ axes with respect to the original $x-y$ axes can also be entered from the lower left corner.

automatically. The results of the new stress matrix will be shown on the lower right corner of the screen. A graphical illustration will also be plotted on the large upper left panel.

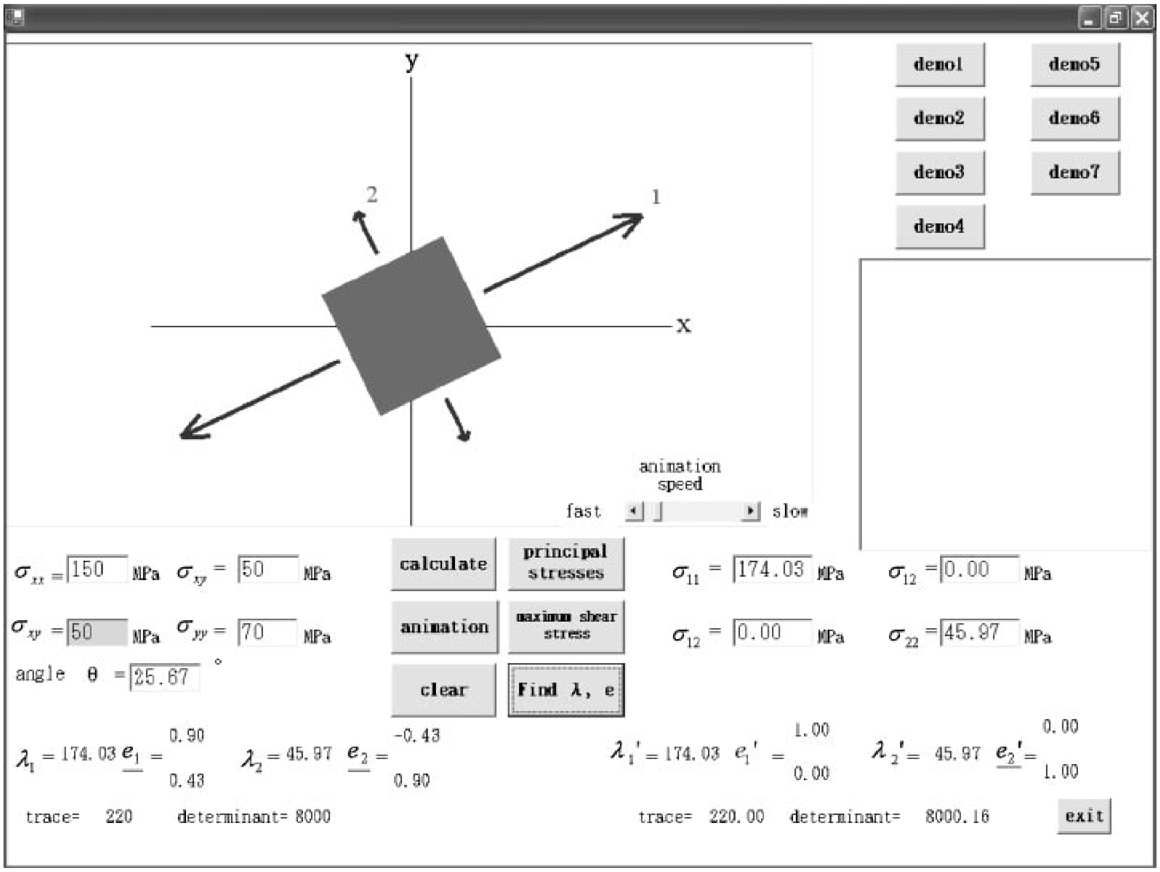
As can be seen from Figure 5, the original stress matrix and the angle in this particular case is selected to be

$$\sigma = \begin{bmatrix} 150 & 50 \\ 50 & 70 \end{bmatrix} (\text{MPa}), \text{ and } \theta = 0^\circ.$$

If we press the "principal stresses" button, the software will calculate the eigenvalues and the corresponding eigenvectors of the original stress matrix σ. The graphical results and the new stress matrix σ' will be shown in Figure 6. For the stress problem, the eigenvalues represent the principal stresses and the eigenvectors represent the directions of the principal axes.

As can be seen from Figure 6, the shear stress is zero if we describe the stress state of the infinitesimal element with respect to the $1-2$ axes. The stress matrix with respect to $1-2$ axes, shown in the lower right quadrant, is a diagonal matrix. The angle is 25.67° in this particular case. That is, when the $1-2$ axes are rotated 25.67° from the $x-y$ axes counterclockwise, the shear stress will be zero; the normal stresses reach maximum or minimum values and are called the principal stresses.

If we press the "Find λ, e" button, the program will calculate the eigenvalues and the corresponding eigenvectors for both of the matrices and will show the numerical values at the bottom of the screen. As can be seen from Figure 6, the two matrices have the same eigenvalues but different eigenvectors. Although the eigenvectors are different in mathematical form, they point to the same direction in real space. For example, the first eigenvector $\underline{e_1} = (0.90, 0.43)^t$ in the $x-y$ coordinate system points in the same direction as $\underline{e_1'} = (1.00, 0.00)^t$ in the $1-2$ coordinate system. The traces and determinants of the two matrices are also shown at the bottom of the screen.



σ_{xx} = 150 MPa σ_{xy} = 50 MPa

σ_{xy} = 50 MPa σ_{yy} = 70 MPa

angle θ = 25.67 °

σ_{11} = 174.03 MPa σ_{12} = 0.00 MPa

σ_{12} = 0.00 MPa σ_{22} = 45.97 MPa

$\lambda_1 = 174.03 \quad \underline{e_1} = \begin{matrix} 0.90 \\ 0.43 \end{matrix} \qquad \lambda_2 = 45.97 \quad \underline{e_2} = \begin{matrix} -0.43 \\ 0.90 \end{matrix}$

$\lambda_1' = 174.03 \quad e_1' = \begin{matrix} 1.00 \\ 0.00 \end{matrix} \qquad \lambda_2' = 45.97 \quad \underline{e_2}' = \begin{matrix} 0.00 \\ 1.00 \end{matrix}$

trace= 220 determinant= 8000

trace= 220.00 determinant= 8000.16

Figure 6 The appearance of program if the "principal stress" and the "Find λ, e" buttons have been pressed. The two stress matrices are similar. The angle is 25.67° in this particular case. The principal stresses are 174.03 and 45.97 MPa.

The value of the angle 25.67° can be calculated from Equation (11):

$$\frac{1}{2}\tan^{-1}\left(\frac{2 \times 50}{150 - 70}\right) \approx 25.67^\circ,$$

or from the eigenvector.

$$\tan^{-1}\left(\frac{0.43}{0.90}\right) \approx 25.67^\circ$$

In this particular case, the principal stresses are 174.03 and 45.97 MPa as shown in Figure 6. The traces and the determinants of the two matrices are the same regardless of the angle θ. Because the trace and determinant do not vary with angle, they are called invariants. The mathematical expressions of the two invariants for this example are:

$$150 + 70 = 174.03 + 45.97$$

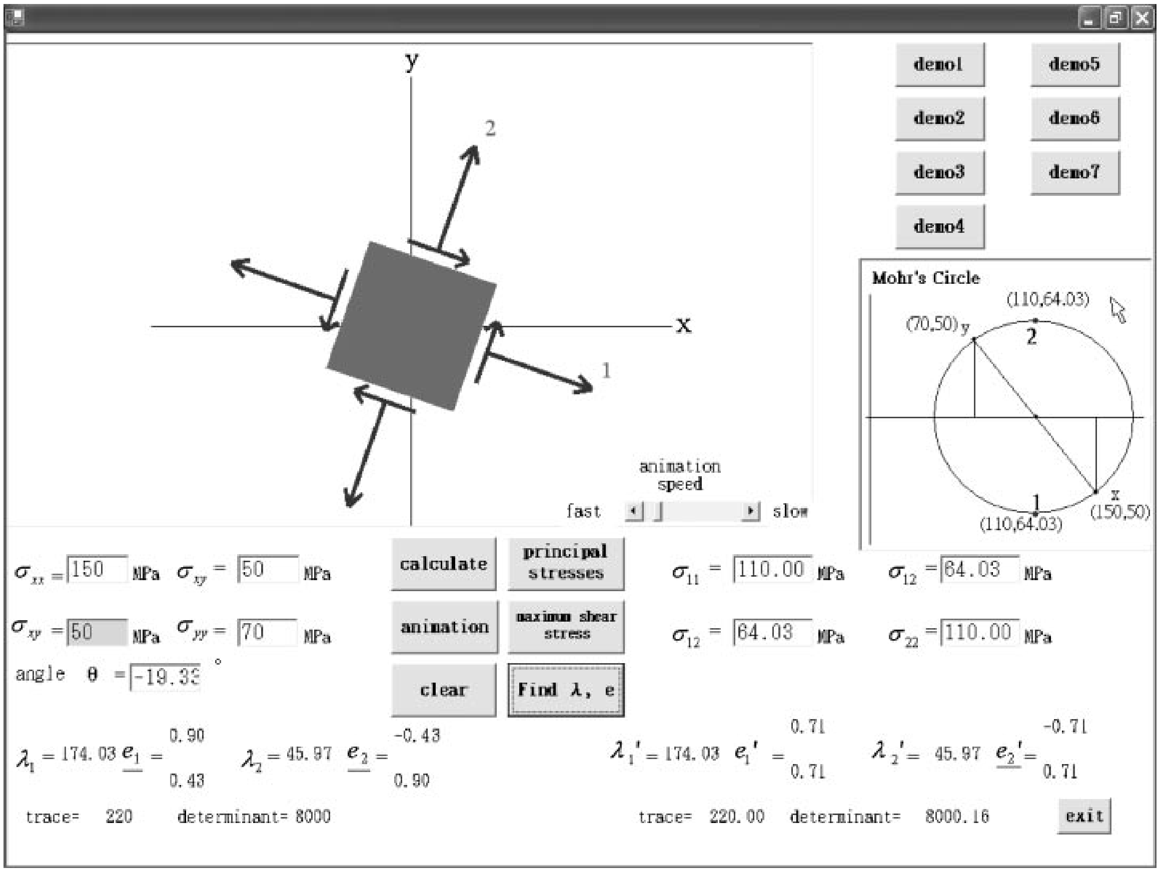
$$(150)(70) - (50)(50) \approx (174.03)(45.97) - 0^2$$

If we press the "maximum shear stress" button, the software will calculate the direction of the axes and the corresponding stresses. The graphical results are also shown in Figure 7. As can been seen from Figure 7, the rotation angle is −19.33°. The shear stress reaches a maximum value of 64.03 MPa. The normal stresses with respect to the 1−2 axes are equal and are both equal to 110 MPa. This value of normal stresses is the average of the two original normal stresses. Thus, when the shear stress reaches its maximum, the normal stresses are equal to the average stress. The mathematical expressions of the two invariants for this example are:

$$150 + 70 = 110 + 110$$

$$(150)(70) - (50)(50) \approx (110)(110) - (64.03)(64.03)$$

If we press the "animation" button, the software will rotate the 1−2 axes automatically. The stresses for every rotational angle will be shown in the lower right quadrant and the graphical presentation will also



σ_{xx} = 150 MPa σ_{xy} = 50 MPa

σ_{xy} = 50 MPa σ_{yy} = 70 MPa

angle θ = -19.33 °

σ_{11} = 110.00 MPa σ_{12} = 64.03 MPa

σ_{12} = 64.03 MPa σ_{22} = 110.00 MPa

$\lambda_1 = 174.03$ $\underline{e_1} = \begin{matrix} 0.90 \\ 0.43 \end{matrix}$ $\lambda_2 = 45.97$ $\underline{e_2} = \begin{matrix} -0.43 \\ 0.90 \end{matrix}$

$\lambda_1' = 174.03$ $e_1' = \begin{matrix} 0.71 \\ 0.71 \end{matrix}$ $\lambda_2' = 45.97$ $\underline{e_2}' = \begin{matrix} -0.71 \\ 0.71 \end{matrix}$

trace= 220 determinant= 8000

trace= 220.00 determinant= 8000.16

Figure 7 The appearance of the program if the "maximum shear stress" button is pressed. The shear stress reaches a maximum value of 64.03 MPa, while the two normal stresses are equal to 110 MPa. The rotational angle is −19.33°. The two stress matrices are similar.

be plotted. The rotational speed can be controlled by the ''animation speed'' controller.

Because some students are not familiar with the topics, they may have difficulties when entering the initial values. Seven ''demo'' buttons are provided in the upper right corner. If the students press one of the seven buttons, the software will enter values of a special case automatically. The seven demo buttons give case of pure tension, pure compression, pure shear, and some general cases.

The Mohr's circle will also be provided in the lower right corner for every special case if the mouse moves over the lower right Mohr's circle plotting panel. Mohr's circle is an alternative way to solve plane stress transformation problems. In Mohr's circle, the horizontal axis represents normal stress while the vertical axis represents shear stress. A shear stress, which tends to rotate the infinitesimal element clockwise, will be drawn above the horizontal axis. As can be seen from Figure 7, a point denoted as ''x'' has coordinates (150,50) on the Mohr's circle. This means that the normal stress is 150 MPa, and the shear stress is 50 MPa, which tends to rotate the infinitesimal element counterclockwise. Therefore, the point ''x'' is drawn below the horizontal axis. If we rotate 38.66° (twice of 19.33°) clockwise from the ''x'' point, we will reach the bottom of the circle, denoted as ''1'' in the Mohr's circle panel, which gives us the normal stress as 110 MPa and shear stress as 64.03 MPa counterclockwise.

STUDENT EXPERIENCE

This software was used to teach courses such as Mechanics of Materials and Linear Algebra. The students used different sets of stress matrix values and rotational angle values to observe the relations among quantities. The preliminary results are quite satisfactory. Most students have very positive responses to this software.

In Mechanics of Materials, students seem to have higher interest levels since a graphical presentation is provided. Last year, when we did not use this software in teaching Mechanics of Materials, the average feedback from the students at the end of the semester was 3.94 on a 5 point scale. After we used this software this year, the feedback was 4.12. Students seem to have a better understanding of plane stresses, principal stresses, principal axes, maximum shear stresses, and transformation of stresses. Then we include the stress transformation problems in the final exam. However, the improvement in the final exam is small. Last year, the average of the final exam was 63.5. This year the average is 64.1. This is probably because we teach the two approaches in solving stress transformation problems. Last year's students concentrated on traditional Equations (7–9). Meanwhile, this year's students study both the traditional approach and linear algebra approach (13), so the improvement is small. Although the difference in exams is minor, the linear algebra approach, Equation (13), is much easier to memorize than Equations (7–9). Probably, this software may help students more in the long term.

In Linear Algebra, students will realize that eigenvalue problems can be applied in stress transformation. Students will have a higher motivation to learn how to solve eigenvalue problems. Before we used this software in teaching Linear Algebra, the feedback from the students at the end of the semester was 3.84 on a 5 point scale. After we used this software, the feedback was 4.22. The students tend to have a better understanding of eigenvalues, eigenvectors, symmetric matrices, similar matrices, orthogonality of eigenvectors, linear transformation, invariants, and diagonalization. Moreover, the improvement in the final exam is quite significant. In our course schedule, eigenvalue problems do not appear in the midterm exam, but they play a major part in the final exam. Last year, the average of the final exam was 65.5. This year, the average was 72.3. This is probably because students think eigenvalue problems are important.

In Taiwan, Students usually take Linear Algebra during their first year and take Mechanics of Materials during their second or third year in university. It is to be noted that students in Taiwan take Linear Algebra from the faculty of their own department, not from the faculty of mathematics. Therefore, students who are taking Linear Algebra in our departments understand that they will have to learn plane stress transformation at a later time.

This program is designed for students in engineering who are studying Mechanics of Materials or Linear Algebra for the first time in the university level. If this program is used to teach students in other fields, such as mathematics, the instructors may have to explain concepts in mechanics very clearly.

It is generally not easy to compare the responses from students because the students are different from year to year. The analysis of responses from students is still quite preliminary. We intend to continue to study students' responses to improve our software.

CONCLUSIONS

This computer program provides a powerful tool for students to understand transformation to plane stress problems. It calculates the stresses with respect to the

1−2 axes, which are rotated counterclockwise through an angle θ about the x−y origin, and shows the visual results on the screen. In addition, students can use animation to observe the change of plane stress. This program can also find the eigenvalues and eigenvectors of the two stress matrices. The program was used in teaching courses, such as Mechanics of Materials and Linear Algebra. The students' motivation was highly increased. The students tended to have higher interest levels and tended to have a better understanding of principal stresses, principal directions, Mohr's circle, eigenvalues, eigenvectors, similar matrices, and invariants.

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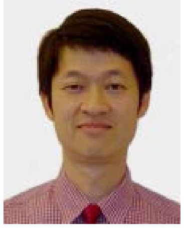
BIOGRAPHIES



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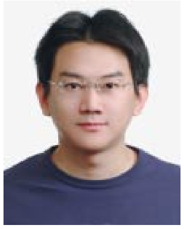

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