Three Notes on the Exponential of a Matrix and Applications

The majority of textbooks on ordinary differential equations use the matrix exponential to solve the linear system (L) $x' = Ax$, A an $n \times n$ constant matrix. In fact, it is well known that the computation of the exponential of a matrix is a difficult numerical and algebraic problem. For the system (L), Putzer's algorithm bypasses much of the difficulty—see Paul Waltman's elegant presentation [3] or W. T. Reid's advanced text [2]. In [1], Ed Leonard presented an alternative method. These three notes are comments, additions, and improvements on his method.

The note of Liz simplifies Leonard's method in that one need only find a basis for the solution space of a related scalar equation, rather than solve a specific set of initial value problems. The note of Kwapisz applies an analogous method to the solution of difference equations which allows one to efficiently calculate powers of matrices. Finally, the note of Hou establishes and exploits a simple relation between the trace of the resolvent and the characteristic polynomial to present a novel derivation of the Leverrier–Faddeev algorithm.

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REFERENCES

[1] I. E. Leonard, *The matrix exponential*, SIAM Rev., 38 (1996), pp. 507–512.
[2] W. T. Reid, *Ordinary Differential Equations*, Wiley, New York, 1971.
[3] P. Waltman, *A Second Course in Elementary Differential Equations*, Academic Press, New York, 1986.