班級：_____學號：_____姓名：_____

國立台灣海洋大學河海工程學系工程數學(二) 2B 班第四次小考參考解答

1. In the course, we determine $\mathcal{L}\{J_0(t)\}$ by considering

$$t^2 J_0''(t) + tJ_0'(t) + t^2 J_0(t) = 0 \tag{1}$$

If we change Eq.(1) into

$$tJ_0''(t) + J_0'(t) + tJ_0(t) = 0 \tag{2}$$

Repeat the process and find the ODE for $\mathcal{L}\{J_0(t)\}$, and $\mathcal{L}\{J_0(t)\}$.

解　令 $y(t) = J_0(t)$, $\mathcal{L}\{J_0(t)\} = Y(s)$ **則可得**

$$\mathcal{L}\{tJ_0''(t)\} = -\frac{d}{ds}Y(S)$$

$$\mathcal{L}\{J_0'(t)\} = sY(s) - 1$$

$$\mathcal{L}\{tJ_0''\} = -\frac{d}{ds}[s^2Y(s) - s] = -s^2Y'(s) - 2sY(s) + 1$$

因此， Eq.(2) **可變為**

$$(s^2+1)Y'(s) + sY(s) = 0 \Rightarrow \qquad Y(s) = \frac{c}{\sqrt{s^2+1}}$$

利用起始值定理可決定常數 c ($y(0)$=1)

$$\lim_{s\to\infty} \frac{c}{\sqrt{s^2+1}} = c = 1 \Rightarrow \qquad J_0(x) = \frac{1}{\sqrt{s^2+1}}$$