Solve the solution of $y''(x)+3y'(x)+2y(x)=0$.

方法1**：**

$$y''(x)+3y(x)+2y(x)=0 \quad \Rightarrow \left(y''(x)+y'(x)\right)+2(y'(x)+y(x))=0$$

$$f(x)=y'(x)+y(x)$$

$$\Rightarrow \left(y''(x)+y'(x)\right)+2(y'(x)+y(x))=f'(x)+2f(x)=0$$

$$f(x)=c_0+c_1x+c_2x^2+c_3x^3+\cdots+c_nx^n+c_{n+1}x^{n+1}+c_{n+2}x^{n+2}+\cdots$$

$$\Rightarrow f'(x)=c_1+2c_2x+\cdots+nc_nx^{n-1}+(n+1)c_{n+1}x^n+(n+2)c_{n+2}x^{n+1}+\cdots$$

$$f'(x)+2f(x)=\left(c_1+2c_2x+\cdots+nc_nx^{n-1}+(n+1)c_{n+1}x^n+(n+2)c_{n+2}x^{n+1}+\cdots\right)+$$
$$2\left(c_0+c_1x+c_2x^2+c_3x^3+\cdots+c_nx^n+c_{n+1}x^{n+1}+c_{n+2}x^{n+2}+\cdots\right)=0$$

$$\Rightarrow c_1+2c_0=0、2c_2+2c_1=0、3c_3+2c_2=0 \;\ldots\; nc_n+2c_{n-1}=0$$

$$\Rightarrow c_n=\left(-\frac{2}{n}\right)c_{n-1}=\left(-\frac{2}{n}\right)\left(-\frac{2}{n-1}\right)c_{n-2}=\cdots=\frac{(-2)^n}{n!}c_0$$

$$\Rightarrow f(x)=c_0\lim_{n\to\infty}\sum_{k=0}^{n}\frac{(-2)^k}{k!}x^k=c_0e^{-2x}$$

$$f(x)=y'(x)+y(x) \;\Rightarrow y'(x)+y(x)=c_0e^{-2x}$$

$$y(x)=a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n+a_{n+1}x^{n+1}+a_{n+2}x^{n+2}+\cdots$$

$$\Rightarrow y'(x)=a_1+2a_2x+\cdots+na_nx^{n-1}+(n+1)a_{n+1}x^n+(n+2)a_{n+2}x^{n+1}+\cdots$$

$$y'(x)+y(x)=\left(a_1+2a_2x+\cdots+na_nx^{n-1}+(n+1)a_{n+1}x^n+(n+2)a_{n+2}x^{n+1}+\cdots\right)+$$
$$\left(a_0+a_1x+a_2x^2+\cdots+a_nx^n+a_{n+1}x^{n+1}+a_{n+2}x^{n+2}+\cdots\right)$$
$$=c_0\left(1+\frac{(-2)^1}{1!}x+\frac{(-2)^2}{2!}x^2\cdots+\frac{(-2)^{n-1}}{(n-1)!}x^{n-1}+\frac{(-2)^n}{n!}x^n+\cdots\right)$$

$$\Rightarrow a_1 + a_0 = c_0、2a_2 + a_1 = c_0 \frac{(-2)^1}{1!} \ \cdots \ na_n + a_{n-1} = c_0 \frac{(-2)^{n-1}}{(n-1)!}$$

$$\Rightarrow a_n = \frac{1}{n}\left(c_0 \frac{(-2)^{n-1}}{(n-1)!} - a_{n-1} \right) = c_0 \frac{(-2)^{n-1}}{n!} - \frac{1}{n}\left(\frac{1}{n-1}\left(c_0 \frac{(-2)^{n-2}}{(n-2)!} \right)\right) + \frac{(-1)^2}{n(n-1)} a_{n-2}$$

$$= c_0 \left((-1)^0 \frac{(-2)^{n-1}}{n!} + (-1)^1 \frac{(-2)^{n-2}}{n!} + \cdots + (-1)^{n-1} \frac{(-2)^0}{n!} \right) + \frac{(-1)^n}{n!} a_0$$

$$= c_0 (-1)^{n-1} \left(\frac{2^{n-1}}{n!} + \frac{2^{n-2}}{n!} + \cdots + \frac{2^0}{n!} \right) + \frac{(-1)^n}{n!} a_0$$

$$= (-c_0) \frac{(-1)^n}{n!} \left(2^{n-1} + 2^{n-2} + 2^{n-3} + \cdots + 2 + 2^0 \right) + \frac{(-1)^n}{n!} a_0$$

$$= (-c_0) \frac{(-1)^n}{n!} \left(\frac{(2^n - 1)}{(2-1)} \right) + \frac{(-1)^n}{n!} a_0$$

$$= (-c_0) \frac{(-1)^n \cdot 2^n}{n!} - (-c_0) \frac{(-1)^n}{n!} + \frac{(-1)^n}{n!} a_0$$

$$= (-c_0) \frac{(-2)^n}{n!} + \frac{(-1)^n}{n!} (a_0 + c_0)$$

$$\Rightarrow y(x) = (-c_0) \lim_{n\to\infty} \sum_{k=0}^{n} \frac{(-2)^k}{k!} + (a_0 + c_0) \lim_{n\to\infty} \sum_{k=0}^{n} \frac{(-1)^k}{k!} = (-c_0) e^{-2x} + (a_0 + c_0) e^{-x}$$

方法 2**：**

$y(x) = e^{sx} \ \Rightarrow y'(x) = se^{sx}$ and $y''(x) = s^2 e^{sx}$

$y''(x) + 3y'(x) + 2y(x) = 0$

$$\Rightarrow (s^2 + 3s + 2)e^{sx} = 0$$

$$\Rightarrow s^2 + 3s + 2 = 0$$

$$\Rightarrow s = -1, \ -2$$

$$\Rightarrow y(x) = e^{-x}, \ e^{-2x}$$

$$\Rightarrow y(x) = me^{-x} + ne^{-2x}$$

方法三：

$$\mathcal{L}\{y(x)\}=Y(s)\quad\Rightarrow\mathcal{L}\{y'(x)\}=sY(s)-y(0)$$

$$\Rightarrow\mathcal{L}\{y''(x)\}=s^2Y(s)-sy(0)-y'(0)$$

$$y''(x)+3y'(x)+2y(x)=\left(s^2Y(s)-sy(0)-y'(0)\right)+3\left(sY(s)-y(0)\right)+2Y(s)=0$$

$$\Rightarrow\left(s^2+3s+2\right)Y(s)-\left(s+3\right)y(0)-y'(0)=0$$

$$\Rightarrow Y(s)=\frac{(s+3)}{s^2+3s+2}y(0)+\frac{1}{s^2+3s+2}y'(0)$$

$$=\frac{(s+2)+1}{(s+1)(s+2)}y(0)+\frac{1}{s^2+3s+2}y'(0)$$

$$=\frac{1}{s+1}y(0)+\left(\frac{1}{s+1}-\frac{1}{s+2}\right)\left(y(0)+y'(0)\right)$$

$$=\frac{1}{s+1}\left(2y(0)+y'(0)\right)+\frac{1}{s+2}\left(-y(0)-y'(0)\right)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}=e^{-x}\ \text{ and }\ \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}=e^{-2x}$$

$$y(x)=\mathcal{L}^{-1}\{Y(s)\}=\left(2y(0)+y'(0)\right)e^{-x}+\left(-y(0)-y'(0)\right)e^{-2x}=me^{-x}+ne^{-2x}$$

$$A: y(x)=me^{-x}+ne^{-2x}$$