**Wronskian(2020)**

|  |  |
| --- | --- |
| $$y''+3y'+2y=0$$ | $$y''+3y'-4y=0$$ |
| $$y\_{1}\left(x\right)=e^{-2x}$$$$y\_{2}\left(x\right)=e^{-x}$$ | $$y\_{1}\left(x\right)=e^{x}$$$$y\_{2}\left(x\right)=e^{-4x}$$ |
| $$W=\left|\begin{matrix}e^{-2x}&e^{-x}\\-2e^{-2x}&-e^{-x}\end{matrix}\right|=e^{-3x}$$ | $$W=\left|\begin{matrix}e^{x}&e^{-4x}\\e^{x}&-4e^{-4x}\end{matrix}\right|=-5e^{-3x}$$ |
| $$W'+3W=0$$$$W\left(x\right)=ke^{-3x}$$ |
| $$y''+a\left(x\right)y'+b\left(x\right)y=0$$$$y\_{1}''+ay\_{1}'+by\_{1}=0$$$$y\_{2}''+ay\_{2}'+by\_{2}=0$$$$y\_{2}\left(y\_{1}''+ay\_{1}'+by\_{1}\right)=0$$$$y\_{1}\left(y\_{2}''+ay\_{2}'+by\_{2}\right)=0$$$$\left(y\_{2}y\_{1}'-y\_{1}y\_{2}'\right)^{'}+a\left(y\_{2}y\_{1}'-y\_{1}y\_{2}'\right)=0$$$$W'+aW=0, W=\left|\begin{matrix}y\_{1}&y\_{2}\\y\_{1}'&y\_{2}'\end{matrix}\right|$$ |

雖然Wronskian看起來只和$ a$有關和$ b$無關，但是$ b$會影響 $y\_{1}與 y\_{2}$。