Envelope by a family of geometry unit

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Clairaut's 微分方程式:

$$y = xy' - \frac{1}{4}y'^2$$

General solution:

$$y = xc - \frac{1}{4}c^2, for\ \ any\ \ c$$

Singular solution $y = y(x)$ by parameter representation

$$x = x(c), y = y(c)$$

Two conditions must be satisfied if $(x(c), y(c))$ is intersection point with the same tangent line

$$y(c) = x(c)\,c - \frac{1}{4}c^2 \tag{1}$$

$$\frac{dy}{dx}\mid_{(x(c),y(c))} = c$$

By considering

$$\frac{dy}{dx}\mid_{(x(c),y(c))} = \frac{dy(c)/dc}{dx(c)/dc}\mid_{(x(c),y(c))} = c \rightarrow y'(c) = cx'(c)$$

Eq.(1) can be differentiated with respect to c, we have

$$y'(c) = x'(c)\,c + x(c) - \frac{1}{2}c$$

Therefore, we have

$$x(c) = \frac{1}{2}c, y(c) = \frac{1}{4}c^2$$

The singular solution is $y = x^2$.

Mohr-Columb criterion

$$(x-a)^2 + y^2 = r^2(a)$$

$$2(x-a) + 2y = 2r(a)$$

The envelope is a straight line.

Monge cone

$$z - z_0 = p(s)(x - x_0) + q(s)(y - y_0)$$

$$0 = p'(s)(x - x_0) + q'(s)(y - y_0)$$

We have the envelope of Monge cone.