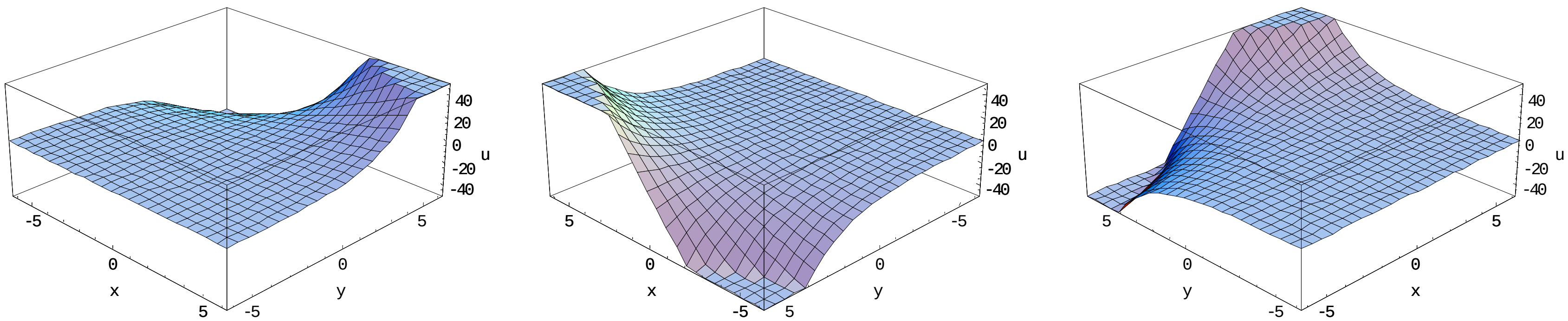
Solve the first-order PDE

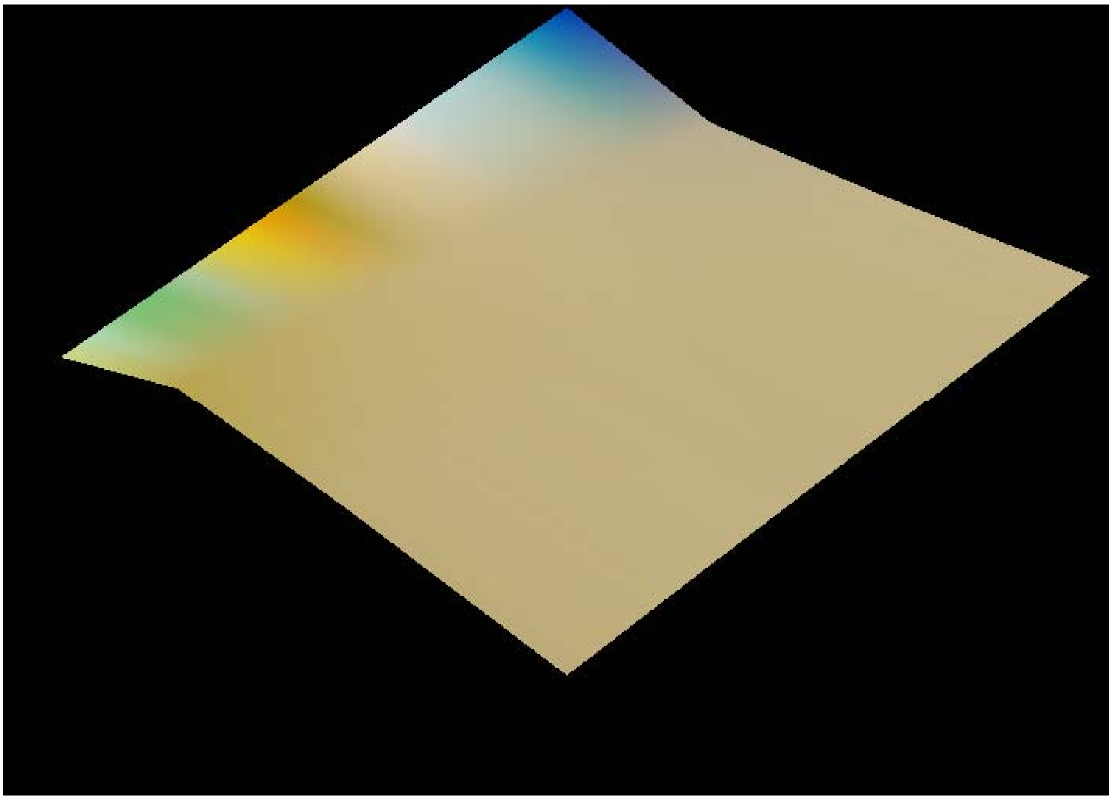
$$x\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}-2u=0\text{, I. C. }\; u(x,0)=x$$

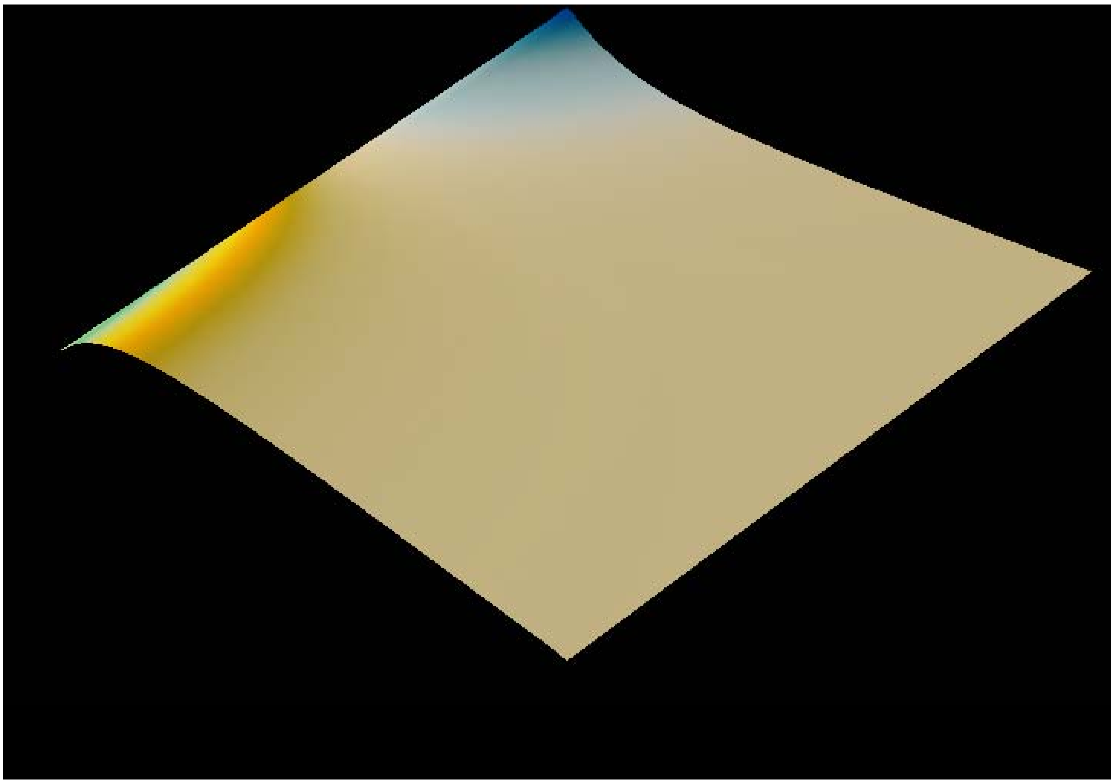
由 $u(x,0)=x$, $\rightarrow\begin{cases}x(0,s)=s\\ y(0,s)=0\\ u(0,s)=s\end{cases}$

$$\because \frac{du}{d\tau}=\frac{du}{dx}\frac{dx}{d\tau}+\frac{du}{dy}\frac{dy}{d\tau}\quad \therefore\begin{cases}\dfrac{du}{d\tau}=2u\ \&\ u(0,s)=s & \rightarrow u=se^{2\tau}\\ \dfrac{dx}{d\tau}=x\ \&\ x(0,s)=s & \rightarrow x=se^{\tau}\quad\text{(Parametric form)}\\ \dfrac{dy}{d\tau}=2\ \&\ y(0,s)=0 & \rightarrow y=2\tau\end{cases}$$

Rearranging $\rightarrow u=xe^{\frac{y}{2}}$ (explicit solution)







Characteristic line

$$\frac{dx}{dy} = \frac{x}{2} \quad \rightarrow \quad y = 2\ln x + c$$