

Quasi-linear first order equation - simple example

I. $u(x, y)$:

II. Quasi-linear PDE:

$$u_x = u_y$$

III. Linear PDE:

$$a(x, y)u_x + b(x, y)u_y = 0$$

where

$$a(x, y) = 1$$

$$b(x, y) = -1$$

$$c(x, y) = 0$$

IV. Assume $z = u(x, y)$

$$z = \text{constant}$$

is the solution of

$$(u_x, u_y, -1) \cdot (a, b, c) = 0$$

V. A curve in parametric form

$$\frac{dx}{dt} = 1, \quad x(0, s) = s$$

$$\frac{dy}{dt} = -1, \quad y(0, s) = 0$$

$$\frac{du}{dt} = 0, \quad z(0, s) = f(s)$$

The solution of the surface in parametric form is

$$x(t, s) = t + s$$

$$y(t, s) = -t$$

$$z(t, s) = f(s)$$

IV. Express s in terms of x and y , we have

$$z = f(x + y)$$

Figure:

IIV. Exercise:

Governing equation:

$$2u_x + 3u_y = 4$$

subjected to

$$\text{case 1. } u(s, s^2) = 5s^2$$

$$\text{case 2. } u(2s, 3s) = 5$$