聯立一階微分方程以 **Laplace transform** 處理

Q: $yu_x-xu_y=3x$　　　Ｉ.Ｃ:**{ $u(x,0)=x^2$ }**　Solve u(x,y)=?

Method①:

$$\begin{cases} yu_x - xu_y = 3x \\ \dfrac{dx}{dt}\dfrac{\partial u}{\partial x} - \dfrac{dy}{dt}\dfrac{\partial u}{\partial y} = \dfrac{\partial u}{\partial t} \end{cases} \Rightarrow \begin{cases} \dfrac{dx}{dt} = y \cdots (1) \\ \dfrac{dy}{dt} = -x \cdots (2) \\ \dfrac{du}{dt} = 3x \cdots (3) \end{cases}$$

$$\frac{(2)}{(1)} \Rightarrow \left(x^2+y^2\right) = s^2 \qquad \frac{(3)}{(2)} \Rightarrow u = -3y + s^2 = -3y + \left(x^2+y^2\right)$$

Method ②: (陳奕安's idea from Laplace transform)

£① ⇒ ŠX-s =Y　　£② ⇒ ŠY=-X

£③ ⇒ ŠU-s^2 =3 X

by <u>克萊瑪 rule or 聯立方程</u>

X= sŠ/(Š²+1) ⇒ £$^{-1}$ X $= x(s,t) = s\cos t$

Y=-s/(Š²+1) ⇒ £$^{-1}$ Y $= y(s,t) = -s\sin t$

U=-3s/(Š²+1)+s^2/Š ⇒ £$^{-1}$U $= u(s,t) = -3s\sin t + s^2$

So, we know $\Rightarrow u(x,y) = -3y + (x^2+y^2)$