Boundary Element Method

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HW No.1

1. For the interior problem, we have derived the dual boundary integral equations(DBIE) for a corner as follows

$$\alpha\, u(x) = C.P.V. \int_B T(s,x)u(s)dB(s) - R.P.V. \int_B U(s,x)t(s)dB(s)$$

$$\alpha\, t^-(x) + sin(\alpha)\, t^+(x) = H.P.V. \int_B M(s,x)u(s)dB(s) - C.P.V. \int_B L(s,x)t(s)dB(s)$$

   Solve the problem 4.
2. For the exterior problem 5, please derive the dual boundary integral equations(DBIE) for a corner and reduce to smooth boundary.
3. Find the differences of the two dual boundary integral equations(DBIE) for a corner between interior and exterior problems.
4. Consider the following problem:
   Governing equation:

$$\nabla^2 u(r,\theta) = 0,\ \ R < r < \infty,\ \ 0 < \theta < 2\pi$$

   Boundary condition:

$$u(r,\theta) = f(\theta),\ for\ r = R$$

   Please derive the Poisson formula for exterior domain.

$$u(\rho,\theta) = \frac{1}{2\pi}\int_0^{2\pi} \frac{\rho^2 - R^2}{R^2 + \rho^2 - 2R\rho\ cos(\theta - \theta')} f(\theta')d\theta'$$

5. Solve the above exterior problem either analytically or numerically for the following B.C.

$$f(\theta) = \pm 1.0,\ +\ for\ \ 0 < \theta < \pi,\ -\ for\ \ \pi < \theta < 2\pi$$

   where the radius is $R = 1$.
6. Plot the potential and potential gradient along the three angles 30, 60, 90 degrees from $\rho = 1$ to $\rho = 5$. Also, plot the normal flux on the circular boundary.
7. Reference exact solution:

$$u(x,y) = \frac{2}{\pi} tan^{-1}\left(\frac{2y}{x^2 + y^2 - 1}\right)$$