

邊界元素法基本教材

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1. For the interior problem, we have derived the dual boundary integral equations(DBIE) for a corner as follows

$$\begin{aligned}\alpha u(x) &= C.P.V. \int_B T(s, x)u(s)dB(s) - R.P.V. \int_B U(s, x)t(s)dB(s) \\ \alpha t^-(x) + \sin(\alpha) t^+(x) &= H.P.V. \int_B M(s, x)u(s)dB(s) - C.P.V. \int_B L(s, x)t(s)dB(s)\end{aligned}$$

Solve the problem 4.

2. For the exterior problem 5, please derive the dual boundary integral equations(DBIE) for a corner and reduce to smooth boundary.
3. Find the differences of the two dual boundary integral equations(DBIE) for a corner between interior and exterior problems.
4. Consider the following problem:

Governing equation:

$$\nabla^2 u(r, \theta) = 0, \quad R < r < \infty, \quad 0 < \theta < 2\pi$$

Boundary condition:

$$u(r, \theta) = f(\theta), \quad \text{for } r = R$$

Please derive the Poisson formula for exterior domain.

$$u(\rho, \theta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\rho^2 - R^2}{R^2 + \rho^2 - 2R\rho \cos(\theta - \theta')} f(\theta') d\theta'$$

5. Solve the above exterior problem either analytically or numerically for the following B.C.

$$f(\theta) = \pm 1.0, \quad + \text{ for } 0 < \theta < \pi, \quad - \text{ for } \pi < \theta < 2\pi$$

where the radius is $R = 1$.

6. Plot the potential and potential gradient along the three angles 30, 60, 90 degrees from $\rho = 1$ to $\rho = 5$. Also, plot the normal flux on the circular boundary.
7. Reference exact solution:

$$u(x, y) = \frac{2}{\pi} \tan^{-1} \left(\frac{2y}{x^2 + y^2 - 1} \right)$$

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【存檔：E:/ctex/course/bemhw2.te】 【建檔:Dec./5/'94】