# 邊界元素法作業

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I. The fundamental solution is defined as follows

$$\frac{d^2U(x,s)}{dx^2} = \delta(x-s)$$

The dual integral equations can be derived by partial integration

$$u(s) = [T(x,s)u(x) - U(x,s)\frac{du(x)}{dx}] \mid_{x=0}^{x=1}$$

$$\frac{du(s)}{ds} = [M(x,s)u(x) - L(x,s)\frac{du(x)}{dx}] \mid_{x=0}^{x=1}$$

The dual integral equations can be changed to

$$u(x) = [T(s,x)u(s) - U(s,x)\frac{du(s)}{ds}] \mid_{s=0}^{s=1}$$

$$\frac{du(x)}{dx} = [M(s,x)u(s) - L(s,x)\frac{du(s)}{ds}] \mid_{s=0}^{s=1}$$

(a). Determine $U(s,x), T(s,x), L(s,x)$ and $M(s,x)$ for $x > s$ and $x < s$.

(b). Plot $U(s,x), T(s,x), L(s,x)$ and $M(s,x)$ versus $x$ for $0 < x, s < 1$.

(c). Determine

$$\lim_{x\to 1^-} U(0,x) = ?$$

$$\lim_{x\to 0^+} T(0,x) = ?$$

$$\lim_{x\to 1^-} L(0,x) = ?$$

$$\lim_{x\to 0^+} M(1,x) = ?$$

(d). Based on the dual integral formulation, solve

$$\frac{d^2u(x)}{dx^2} = 0, \ \textit{subject to}\ u(0) = 0,\ u'(1) = 1.$$

Is it necessary to use $L, M$ equation for the solution ?

(e). Prooof of symmetry and transpose symmetry for the four kernel functions.

$$U(s,x) = U(x,s)$$

$$T(s,x) = L(x,s)\ \ or\ \ T(s,x) = -L(x,s)$$

$$M(s,x) = M(x,s)$$