邊界元素法作業

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I. The fundamental solution is defined as follows

$$\nabla^2 U(x,s) = \delta(x-s)$$

The dual integral equations can be derived

$$2\pi u(s) = \int_B \{T(x,s)u(x) - U(x,s)t(x)\} dB(x)$$

$$2\pi t(s) = \int_B \{M(x,s)u(x) - L(x,s)t(x)\} dB(x)$$

The dual integral equations can be changed to

$$2\pi u(x) = \int_B \{T(s,x)u(s) - U(s,x)t(s)\} dB(s)$$

$$2\pi t(x) = \int_B \{M(s,x)u(s) - L(s,x)t(s)\} dB(s)$$

(a). Determine $U(s,x)$ except the method in course (Fourier Transform or any other method).

(b). Plot $U(s,x), T(s,x), L(s,x)$ and $M(s,x)$ versus x in contour form and 3-D plot for fixed $s=(0,0)$.

(c). Determine the order of singularity $O(\epsilon)$ for $U(s,x), T(s,x), L(s,x)$ and $M(s,x)$ as $x \to s$ by setting $s = x + \epsilon(cos(\theta), sin(\theta))$.

(d). Prooof of symmetry and transpose symmetry for the four kernel functions.

$$U(s,x) = U(x,s)$$

$$T(s,x) = L(x,s) \;\; or \;\; T(s,x) = -L(x,s)$$

$$M(s,x) = M(x,s)$$

(e). Prooof of the following identities.

$$2\pi = \int_B \{T(s,x)\} dB(s)$$

$$0 = \int_B \{M(s,x)\} dB(s)$$

(f). Find the dependence of normal vectors in the four kernels.