

## 邊界元素法作業

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I. Dual integral equations for complex variables on a domain point:

$$W(z) = u(x, y) + iv(x, y) = \frac{1}{2\pi i} \int_B \frac{W(t)}{t - z} dt \quad (1)$$

$$W'(z) = \frac{1}{2\pi i} \int_B \frac{W(t)}{(t - z)^2} dt \quad (2)$$

Dual integral equations for complex variables on a boundary point:

$$W(z) = u(x, y) + iv(x, y) = \frac{1}{2} W(z) + \frac{1}{2\pi i} CPV \int_B \frac{W(t)}{t - z} dt \quad (3)$$

$$W'(z) = \frac{1}{2\pi i} HPV \int_B \frac{W(t)}{(t - z)^2} dt \quad (4)$$

- (a). If  $W(z)$  is analytic function, then  $u, v$  are Cauchy Riemann pairs.
- (b). Based on Eqs.(1) and (2), derive the dual integral equations for real variables on a domain point

$$2\pi u(x) = \int_B \{T(s, x)u(s) - U(s, x)t(s)\} dB(s)$$

$$2\pi t(x) = \int_B \{M(s, x)u(s) - L(s, x)t(s)\} dB(s)$$

Based on Eqs.(3) and (4), derive the dual integral equations for real variables on a boundary point

$$\pi u(x) = CPV \int_B T(s, x)u(s) dB(s) - RPV \int_B U(s, x)t(s) dB(s)$$

$$\pi t(x) = HPV \int_B M(s, x)u(s) dB(s) - CPV \int_B L(s, x)t(s) dB(s)$$

- (c). Discuss the Cauchy principal value as  $z$  approaches  $B$ .
- (d). Discuss the Hadamard principal value as  $z$  approaches  $B$ .
- (e). Proof of the following identities.

$$2\pi = \int_B \{T(s, x)\} dB(s)$$

$$0 = \int_B \{M(s, x)\} dB(s)$$