邊界元素法1999 第十一次作業

1. In the course, the fundamental solution $U(x,s)$ satisfies

$$\frac{d^2U(x,s)}{dx^2} = \delta(x-s) \tag{1}$$

where

$$U(x,s) = \begin{cases} \frac{1}{2}(x-s), & x > s, \\ -\frac{1}{2}(x-s), & x < s \end{cases} \tag{2}$$

the boundary integral equation can be obtained as

$$u(s) = \frac{\partial U(x,s)}{\partial x}u(x)\Big|_0^1 - U(x,s)\frac{du(x)}{dx}\Big|_0^1 \tag{3}$$

By approaching the field point s to 0^+ and 1^-, we have derived the stiffness matrix of $[K]$ such that

$$[K]\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \end{bmatrix}$$

where $P_0 = -t(0)$ and $P(1) = t(1)$.

2. It is interesting to find that $U_c(x,s) = U(x,s) + ax + b$ also satisfies Eq.(1) to be an auxilliary system, where a and b are arbitrary constants, please reconstruct the stiffness matrix using $U_c(x,s)$ instead of $U(x,s)$ in Eq.(3), i.e.,

$$u(s) = \frac{\partial U_c(x,s)}{\partial x}u(x)\Big|_0^1 - U_c(x,s)\frac{du(x)}{dx}\Big|_0^1 \tag{4}$$

where the stiffness matrix $[K]$ satisfy

$$[K]\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} -\frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \end{bmatrix}$$

and compare the result in the course by using any a and b.

3. Is it possible that the matrix $[U_{ab}]$ in

$$[U_{ab}]\{t\} = [T_{ab}]\{u\}$$

can not be invertible for some combinations of a and b. If yes, can you explain the phenomenon ?

4. Is it possible to derive the free-free flexibility matrix $[F]$ such that

$$[F]\mathbf{P} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} P(0) \\ P(1) \end{bmatrix} = \begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \end{bmatrix}$$

Ref:

C. A. Fellipa, K. C. Park and M. R. J. Filho, The construction of free-free flexibility matrices as generalized inverses, *Computers & Structure*, Vol.68, pp.41-48, 1998.