邊界元素法1999 第十六次作業

1. In the previous homework, we have plotted the fundamental solution, $U(s,x)$, in the closed and degenerate forms as follows:
   Closed form:
   $$U(s,x) = ln(r)$$
   Degenerate form:
   $$U(s,x) = \ln r = \ln \sqrt{(\rho\cos(\phi) - R\cos(\theta))^2 + (\rho\sin(\phi) - R\sin(\theta))^2}$$
   $$= \begin{cases} U^i(s,x) = \ln R - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{\rho}{R})^m \cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = \ln \rho - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{R}{\rho})^m \cos(m(\theta-\phi)),\ \rho > R \end{cases}$$
   $$T(s,x) = \frac{\partial U(s,x)}{\partial n(s)}$$
   $$= \begin{cases} T^i(s,x) = \frac{1}{R} + \sum_{m=1}^{\infty} \frac{\rho^m}{R^{m+1}} \cos(m(\phi-\theta)),\ R > \rho \\ T^e(s,x) = -\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \cos(m(\phi-\theta)), \quad \rho > R \end{cases}$$
   where $s = (R,\theta)$, $x = (\rho,\phi)$ and $r = \mid x - s \mid$.
2. Please derive the closed forms for $L(s,x)$ and $M(s,x)$.
3. Please derive the degenerate forms for $L(s,x)$ and $M(s,x)$.
4. Plot the contours for $L(s,x)$ and $M(s,x)$ using (2) and (3), explain why the results are not the same.
   Closed form:
   $$(s_1, s_2) = (0,0)$$
   $$\mathbf{n}(x) = (0,1),\ \ \mathbf{n}(s) = (0,1)$$
   Degenerate form:
   $$(R,\theta) = (1, 45^o)$$
   $$\mathbf{n}(x) = \mathbf{e}_\rho,\ \ \mathbf{n}(s) = \mathbf{e}_R$$

References

[1] 陳正宗與洪宏基，邊界元素法，第二版，新世界出版社，台北，頁 90 與 214，1992。