邊界元素法期末考 2001 by Prof. J. T. Chen

考試時間 — 9:20 to 11:00, May 31, 2001 考試方式 — Open book

1. Explain the following items. (30%)
   (1). dual integral equations and dual BEM
   (2). Hadamard principal value and Cauchy principal value
   (3). kernel function, fundamental solution and Green's function
   (4). degenerate boundary, degenerate kernel and degenerate scale
   (5). single, double layer and volume potentials
   (6). two-point function

2. In the stage of developing dual BEM program, how can you check the U, T, L and M matrices ? (5%) Do the techniques fail for the problems with degenerate boundary ? (5%) Any other alternatives to determine the diagonal terms for M matrices free from using the HPV concept ? (5%) How can you check the equilibrium condition by $U^{-1}T$ or $L^{-1}M$ for the problems with normal boundary ? (5%) Can the check method be applied to the Helmholtz equation ? Why ? (5%)

3. Give comments on direct and indirect BEMs ? (10%)

4. Please write down the symmetry and transpose symmetry properties for the four kernels $(U(s,x), T(s,x), L(s,x), M(s,x))$ in the dual formulation. (10 %)

5. Please derive the fundamental solution of a beam, i.e.,

   $$\frac{d^4U(x,s)}{dx^4} = \delta(x-s)$$

   by any method you can. (10 %)

6. The force between the two masses, M and m is

   $$\mathbf{F} = \frac{-GMm}{r^2}\hat{\mathbf{r}}$$

   where r is the distance between the two masses. Now consider the mass M as a concentrated mass $1\,g$ and the mass $m = \rho\, ds$ as a uniform distributed mass with density ρ per unit length. If the distributed mass $(\rho\, ds)$ locates along $s=-1$ to $s=1$ and the concentrated mass locates at $x=a$, find the total force between the concentrated mass and the distributed mass for the three cases, $(a<-1, -1<a<1, a>1)$. (10 %) Assume that the point locates at $(0,\epsilon)$, find the forces (x,y,z) for three cases, $\epsilon = 0^-, 0, 0^+$. (10 %) Also, please determine the equivalent locations of the lumped mass for all the cases. (5 %) Give comments by using the Hadamard pricipal value. (5 %) (Hint: Kellog book, pp.4-6)