程式 102 Elasticity problems with holes

Governing equation:

$$(\lambda + G)\nabla(\nabla \cdot \vec{u}(x)) + G\nabla^2 \vec{u}(x) = 0, \quad x \in \Omega$$

Boundary condition:

$\vec{u}$ ⟶ displacement specified

$\vec{t}$ ⟶ traction specified

Fundamental solution:

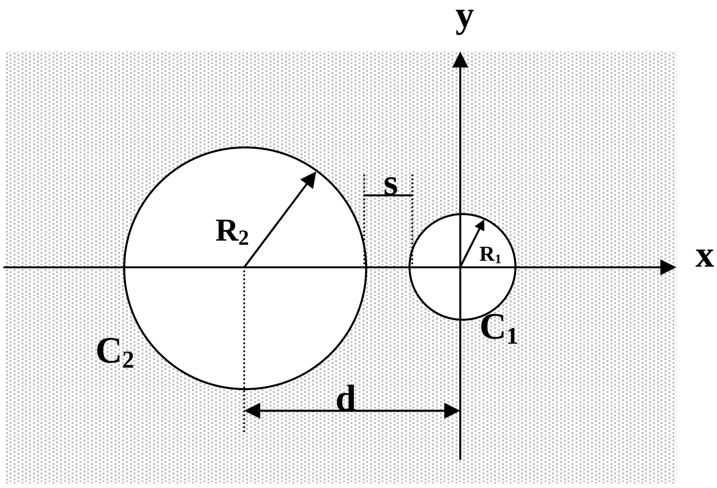
Kelvin solution $U_{ki}(s, x)$

$$U_{ki}(s,x) = \frac{-1}{8\pi G(1-\nu)}[(3-4\nu)\delta_{ki}\ \ln(r) - \frac{y_i y_k}{r^2}] \quad \text{(2-D)}$$

$$U_{ki}(s,x) = \frac{1}{16\pi G(1-\nu)}\frac{1}{r}[(3-4\nu)\delta_{ki} + \frac{y_i y_k}{r^2}] \quad \text{(3-D)}$$

Formulation:

1. Fourier series expansion



$$u_1(s) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta + \sum_{n=1}^{\infty} b_n \sin n\theta$$

$$u_2(s) = \bar{a}_0 + \sum_{n=1}^{\infty} \bar{a}_n \cos n\theta + \sum_{n=1}^{\infty} \bar{b}_n \sin n\theta$$

$$t_1(s) = c_0 + \sum_{n=1}^{\infty} c_n \cos n\theta + \sum_{n=1}^{\infty} d_n \sin n\theta$$

$$t_2(s) = \bar{c}_0 + \sum_{n=1}^{\infty} \bar{c}_n \cos n\theta + \sum_{n=1}^{\infty} \bar{d}_n \sin n\theta$$

2. Null-field integral equations

$$0 = \int_B U_{ij}(s,x)t_i(s)dB(s) - \int_B T_{ij}(s,x)u_i(s)dB(s), \quad x \in \Omega^e$$

$$0 = \int_B L_{ij}(s,x)t_i(s)dB(s) - \int_B M_{ij}(s,x)u_i(s)dB(s), \quad x \in \Omega^e$$

3. Boundary integral equations for the interior potential

References:

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2. S. L. Crouch and S. G. Mongilevskaya, On the use of Somigliana's formula and Fourier series for elasticity problems with circular boundaries, International Journal for Numerical Methods in Engineering, Vol.58, pp.0537-578, 2003.
3. R. A. W. Haddon, Stresses in an infinite plate with two unequal circular holes, Quart. Journ. Mech. and Applied Math.Vol.XX, Pt.3, pp.277-291, 1967.