Using LM equation (direct LM) and LM (indirect-double layer potential approach) to rederive the solution

$$2\pi t(x)=\int_B M(s,x)u(s)dB(s)-\int_B L(s,x)t(s)dB(s),\ x\in D$$

$$0=\int_B M(s,x)u(s)dB(s)-\int_B L(s,x)t(s)dB(s),\ x\to B^+\notin D$$

$$u(x)=\int_{B^+} T(s,x)\phi(s)dB(s),\ x\in D$$

ANS $0=\int_B M(s,x)u(s)dB(s)-\int_B L(s,x)t(s)dB(s),\ x\to B^+\notin D$

$$L(s,x)=\begin{cases} L^i(R,\theta;\rho,\phi)=-\sum_{m=1}^{\infty}(\frac{\rho^{m-1}}{R^m})\cos m(\theta-\phi) & R>\rho \\ L^e(R,\theta;\rho,\phi)=\frac{1}{\rho}+\sum_{m=1}^{\infty}(\frac{R^m}{\rho^{m+1}})\cos m(\theta-\phi) & \rho>R \end{cases}$$

$$M(s,x)=\begin{cases} M^i(R,\theta;\rho,\phi)=\sum_{m=1}^{\infty}(\frac{m\rho^{m-1}}{R^{m+1}})\cos m(\theta-\phi) & R>\rho \\ M^e(R,\theta;\rho,\phi)=\sum_{m=1}^{\infty}(\frac{mR^{m-1}}{\rho^{m+1}})\cos m(\theta-\phi) & \rho>R \end{cases}$$

$$\to 0=\int_B[\sum_{m=1}^{\infty}(\frac{mR^{m-1}}{\rho^{m+1}})\cos m(\theta-\phi)]\cos 2\theta Rd\theta$$

$$-\int_B[\frac{1}{\rho}+\sum_{m=1}^{\infty}(\frac{R^m}{\rho^{m+1}})\cos m(\theta-\phi)][p_0+\sum_{n=1}^{\infty}p_n\cos(n\phi)+q_n\sin(n\phi)]Rd\theta$$

$$=\frac{2\pi}{\rho^3}\cos 2\phi-[\frac{2\pi}{\rho}p_0+\pi\sum_{m=1}^{\infty}(\frac{1}{\rho^{m+1}})p_m\cos m\phi+q_m\sin m\phi]$$

比較係數 $\to p_0=0,\ q_m=0,\ p_m=0,\ m\neq 2\quad p_2=2$

$$2\pi t(x)=\int_B[\sum_{m=1}^{\infty}(\frac{m\rho^{m-1}}{R^{m+1}})\cos m(\theta-\phi)]\cos 2\theta Rd\theta$$

$$-\int_B[-\sum_{m=1}^{\infty}(\frac{\rho^{m-1}}{R^m})\cos m(\theta-\phi)]2\cos 2\theta Rd\theta$$

$$=2\pi\rho\cos 2\phi+2\pi\rho\cos 2\phi$$

$$t(x)=2\rho\cos 2\phi$$

$$u(x)=\int_{B^+} T(s,x)\phi(s)dB(s),\ x\in D$$

$$T(s,x)=\begin{cases} T^i(R,\theta;\rho,\phi)=\frac{1}{R}+\sum_{m=1}^{\infty}(\frac{\rho^m}{R^{m+1}})\cos m(\theta-\phi) & R>\rho \\ T^e(R,\theta;\rho,\phi)=-\sum_{m=1}^{\infty}(\frac{R^{m-1}}{\rho^m})\cos m(\theta-\phi) & \rho>R \end{cases}$$

$$u(x)=\int_{B^+} T(s,x)\phi(s)dB(s),\ x\in D$$

$$\rightarrow u(x)=\int_0^{2\pi}[\frac{1}{R}+\sum_{m=1}^{\infty}(\frac{\rho^m}{R^{m+1}})\cos m(\theta-\phi)][p_0+\sum_{n=1}^{\infty}p_n\cos(n\phi)+q_n\sin(n\phi)]Rd\theta$$

$$=2\pi p_0+\pi\sum_{m=1}^{\infty}\rho^m[p_m\cos(m\phi)+q_m\sin(m\phi)]$$

$$\rightarrow u(1,\phi)=\cos(2\phi)=2\pi p_0+\pi\sum_{m=1}^{\infty}\rho^m[p_m\cos(m\phi)+q_m\sin(m\phi)]$$

比較係數 $\rightarrow p_0=0, q_m=0,\ p_m=0, m\neq 2 \quad p_2=\frac{1}{\pi}$

$$u(x)=\rho^2\cos(2\phi)$$