Using degenerate kernel to find $U_{31}, T_{31}, L_{31}, M_{31}$

and mathematics to determine $U_{33}, T_{33}, L_{33}, M_{33}$

ANS $U_{31} = 2\int_0^{\frac{1}{2}} \ln\rho - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{R}{\rho})^m \cos m(\theta-\phi)\, dR$

$$= 2[R\ln\rho - \sum_{m=1}^{\infty} \frac{1}{m(m+1)} \frac{R^{m+1}}{\rho^m} \cos m\phi \Big|_0^{\frac{1}{2}}]$$

$$= -2\sum_{m=1}^{30} \frac{1}{m(m+1)} (\frac{1}{2})^{m+1} \cos\frac{\pi}{2} m = 0.038867$$

$$T_{31} = \frac{\partial U}{\partial s_2} = \sin\theta \frac{\partial}{\partial R}(U) + \frac{\cos\theta}{R} \frac{\partial}{\partial\theta}(U) = \frac{1}{R}\frac{\partial}{\partial\theta}(U)$$

$$= -\frac{1}{R}\sum_{m=1}^{\infty} \frac{1}{m} (\frac{R}{\rho})^m - m\sin m(\theta-\phi)$$

$$= 2\int_0^{\frac{1}{2}} \sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \sin m\phi\, dR$$

$$= 2\sum_{m=1}^{30} \frac{1}{m} (\frac{1}{2})^m \sin\frac{\pi}{2} m = 0.927295$$

$$L_{31} = \frac{\partial U}{\partial x_2} = \sin\phi \frac{\partial}{\partial\rho}(U) + \frac{\cos\phi}{\rho} \frac{\partial}{\partial\phi}(U) = \frac{\partial}{\partial\rho}(U)$$

$$= \frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{R^m}{\rho^{m+1}} \cos m\phi$$

$$= 2\int_0^{\frac{1}{2}} \frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{R^m}{\rho^{m+1}} \cos m\phi\, dR$$

$$= 2[\frac{1}{2} + \sum_{m=1}^{30} \frac{1}{m+1} (\frac{1}{2})^{m+1} \cos\frac{\pi}{2} m] = 0.927295$$

$$M_{31} = \frac{\partial L}{\partial s_2} = \sin\theta \frac{\partial}{\partial R}(L) + \frac{\cos\theta}{R} \frac{\partial}{\partial\theta}(L) = \frac{1}{R}\frac{\partial}{\partial\theta}(L)$$

$$= \sum_{m=1}^{\infty} -m \frac{R^{m-1}}{\rho^{m+1}} \sin m(\theta-\phi)$$

$$= 2\int_0^{\frac{1}{2}} \sum_{m=1}^{\infty} -m \frac{R^{m-1}}{\rho^{m+1}} \sin m\phi\, dR$$

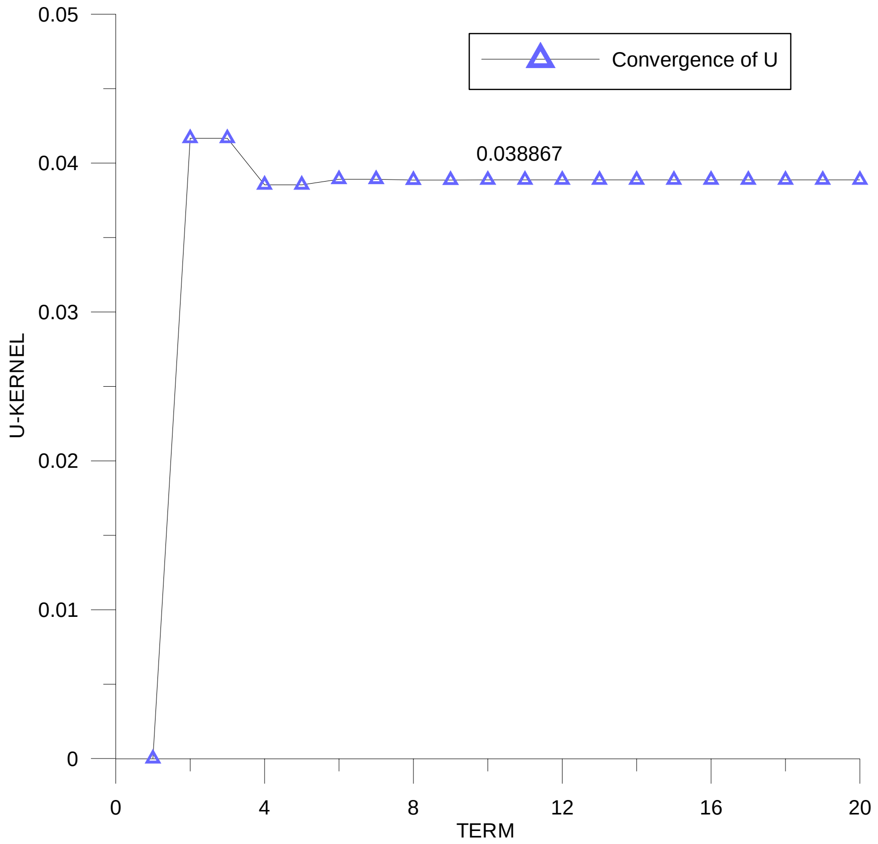
$$= 2[\sum_{m=1}^{30} -(\frac{1}{2})^m \sin\frac{\pi}{2} m] = -0.8$$

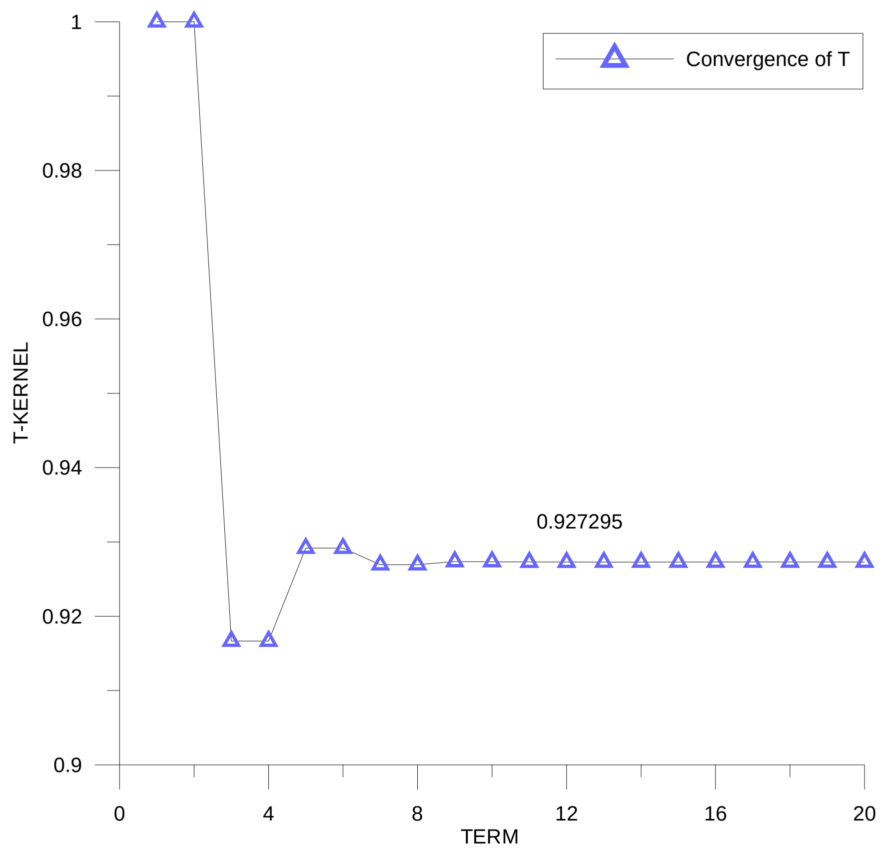
$$U_{33} = \text{Limit}[\int_0^1 \ln[\sqrt{(0.5-s)^2+\varepsilon^2}]\, ds, \varepsilon \to 0] = -1.69315$$

$$T_{33} = \text{Limit}[-\int_0^1 \frac{\varepsilon}{(0.5-s)^2+\varepsilon^2}\, ds,\ \varepsilon \to 0] = -3.14159$$

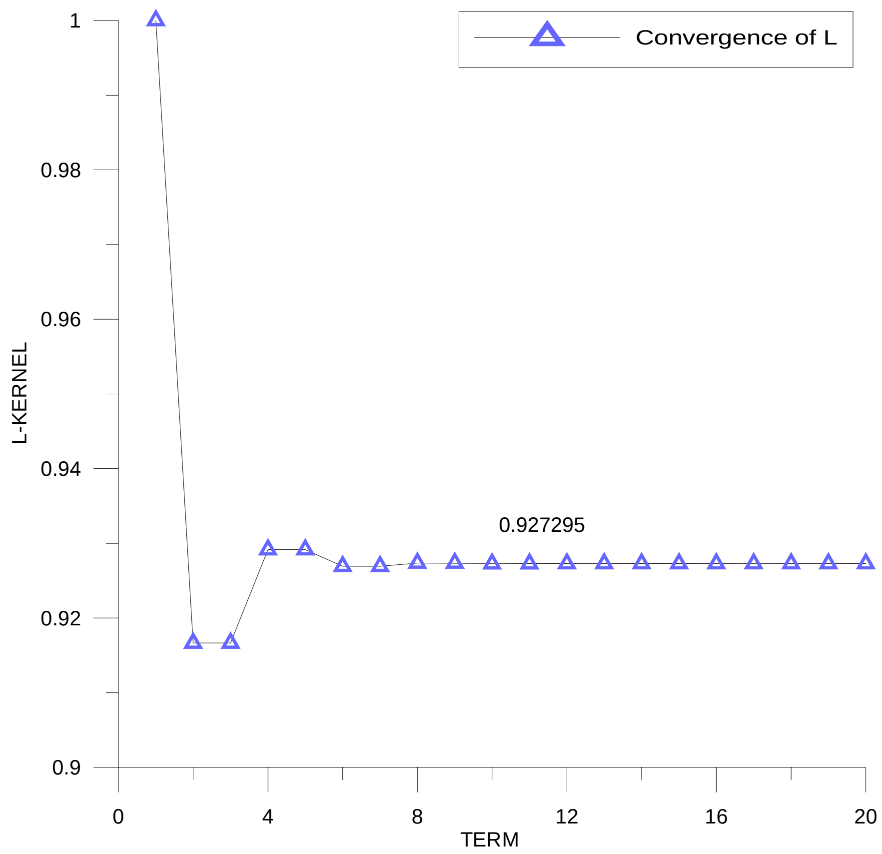
$$L_{33} = \text{Limit}[\int_0^1 \frac{\varepsilon}{(0.5-s)^2+\varepsilon^2}\, ds,\ \varepsilon \to 0] = 3.14159$$

$$M_{33} = \text{Limit}[\int_0^1 \frac{2\varepsilon^2}{((0.5-s)^2+\varepsilon^2)^2} - \frac{1}{(0.5-s)^2+\varepsilon^2}\, ds,\ \varepsilon \to 0] = 4$$

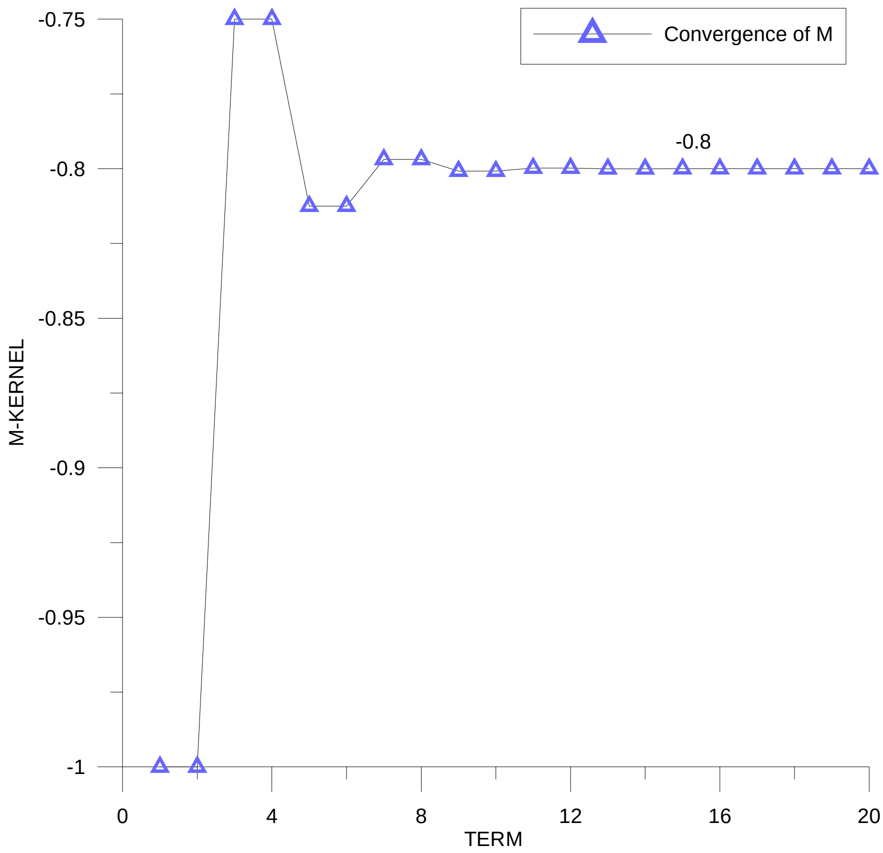

Convergence of U
0.038867
U-KERNEL
TERM
0.05
0.04
0.03
0.02
0.01
0
0
4
8
12
16
20



Convergence of T
0.927295
T-KERNEL
TERM
1
0.98
0.96
0.94
0.92
0.9
0
4
8
12
16
20



Convergence of L
0.927295
L-KERNEL
TERM
1
0.98
0.96
0.94
0.92
0.9
0
4
8
12
16
20



Convergence of M
-0.8
-0.75
-0.8
-0.85
-0.9
-0.95
-1
M-KERNEL
0
4
8
12
16
20
TERM