

$$y_1 = 0.5 - s$$

$$y_2 = 1$$

$$\mathbf{n} = (n_1, n_2) = (0, -1)$$

$$\bar{\mathbf{n}} = (\bar{n}_1, \bar{n}_2) = (0, 1)$$

$$U(s, x) = \ln |s - x| = \ln(r) = \ln \left(\sqrt{(0.5 - s)^2 + 1^2} \right)$$

$$U_{31} = \int_0^1 \ln \left(\sqrt{(s - 0.5)^2 + 1^2} \right) ds = 0.038867$$

$$T(s, x) = \frac{-y_i n_i}{r^2} = \frac{1}{(s - 0.5)^2 + 1^2}$$

$$T_{31} = \int_0^1 \frac{1}{(s - 0.5)^2 + 1^2} ds = 0.927295$$

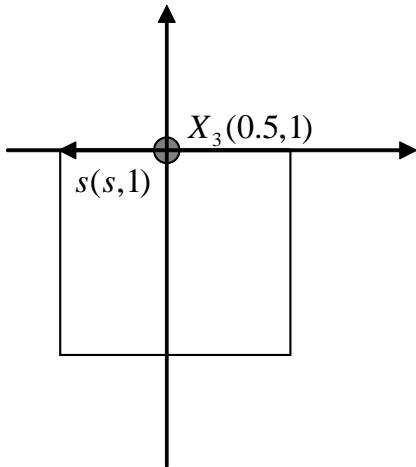
$$L(s, x) = \frac{y_i \bar{n}_i}{r^2} = \frac{1}{(s - 0.5)^2 + 1^2}$$

$$L_{31} = \int_0^1 \frac{1}{(s - 0.5)^2 + 1^2} ds = 0.927295$$

$$M(s, x) = \frac{2 y_i n_i y_j \bar{n}_j}{r^4} - \frac{n_i \bar{n}_i}{r^2} = \frac{-2}{((s - 0.5)^2 + 1^2)^2} + \frac{1}{(s - 0.5)^2 + 1^2}$$

$$M_{31} = \int_0^1 \left(\frac{-2}{((s - 0.5)^2 + 1^2)^2} + \frac{1}{(s - 0.5)^2 + 1^2} \right) ds = -0.8$$

Null



$$U(s, x) = \ln(R) - \sum_{m=1}^{\infty} \left(\frac{1}{m} \right) \left(\frac{\rho}{R} \right)^m \cos[m(\theta - \phi)]$$

$$(\rho, \phi) = 0, \theta = 0$$

$$U(s, x) = \ln(R)$$

$$U_{33} = U_{31} = 2 \int_0^{0.5} \ln(R) dR = -1.69315$$

$$\begin{aligned} T(s, x) &= \frac{\partial U}{\partial s_2} = \sin[\theta] \frac{\partial U}{\partial R} + \frac{\cos[\theta]}{R} \frac{\partial U}{\partial \theta} \\ &= \sin[\theta] \left(\frac{1}{R} + \sum_{m=1}^{\infty} \frac{\rho^m}{R^{m+1}} \cos[m(\theta - \phi)] \right) + \frac{\cos[\theta]}{R} \sum_{m=1}^{\infty} \left(\frac{\rho^m}{R^m} \right) \sin[m(\theta - \phi)] \end{aligned}$$

$$(\rho, \phi) = 0, \theta = 0$$

$$T(s, x) = 0$$

$$\begin{aligned} L(s, x) &= \frac{\partial U}{\partial x_2} = \sin[\phi] \frac{\partial U}{\partial \rho} + \frac{\cos[\phi]}{\rho} \frac{\partial U}{\partial \phi} \\ &= \sin[\phi] \left(- \sum_{m=1}^{\infty} \frac{\rho^{m-1}}{R^m} \cos[m(\theta - \phi)] + \frac{1}{\rho} \cos[\phi] \left(- \sum_{m=1}^{\infty} \left(\frac{\rho}{R} \right)^m \sin[m(\theta - \phi)] \right) \right) \end{aligned}$$

$$(\rho, \phi) = 0, \theta = 0$$

$$L(s, x) = 0$$

$$\begin{aligned} M(s, x) &= \frac{\partial L}{\partial s_2} = \sin[\theta] \frac{\partial L}{\partial R} + \frac{\cos[\theta]}{R} \frac{\partial L}{\partial \theta} \\ &= \\ &\sin[\phi] \sin[\theta] \left(\sin[\phi] \sum_{m=1}^{\infty} m \frac{\rho^{m-1}}{R^{m+1}} \cos[m(\theta - \phi)] \right) + \frac{\sin[\theta] \cos[\phi]}{\rho} \left(\sum_{m=1}^{\infty} m \frac{\rho^m}{R^{m+1}} \sin[m(\theta - \phi)] \right) \\ &+ \frac{\cos[\theta]}{R} \left(\sin[\phi] \sum_{m=1}^{\infty} m \frac{\rho^{m-1}}{R^m} \sin[m(\theta - \phi)] \right) + \\ &\frac{1}{R} \cos[\theta] \left(\frac{1}{\rho} \cos[\phi] \left(- \sum_{m=1}^{\infty} m \left(\frac{\rho}{R} \right)^m \cos[m(\theta - \phi)] \right) \right) \end{aligned}$$

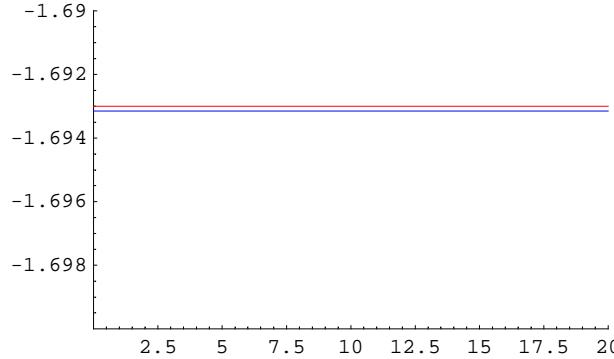
$$(\rho, \phi) = 0, \theta = 0$$

$$\begin{aligned}
& 2 \int_{\varepsilon}^{0.5} \frac{1}{\rho R} \left(- \sum_{m=1}^{\infty} m \left(\frac{\rho}{R} \right)^m \right) dR = 2 \int_{\varepsilon}^{0.5} \left(- \sum_{m=1}^{\infty} m \frac{\rho^{m-1}}{R^{m+1}} \right) dR \\
& = 2 \int_{\varepsilon}^{0.5} \frac{-1}{R^2} dR = \frac{2}{R} \Big|_{\varepsilon}^{0.5} = 4 - \frac{2}{\varepsilon} \\
M_{33} & = 2 \int_{\varepsilon}^{0.5} \frac{1}{\rho R} \left(- \sum_{m=1}^{\infty} m \left(\frac{\rho}{R} \right)^m \right) dR + \frac{2}{\varepsilon} = 4
\end{aligned}$$

```

dat1 := Table[{c, 2 ∫₀⁰·⁵ (Log[e, R]) dR}, {c, 0, 20}]
dat2 := Table[{d, -1.693}, {d, 0, 20}]
g3 := ListPlot[dat1, PlotJoined → True, PlotStyle → {RGBColor[0, 0, 1]}]
g4 := ListPlot[dat2, PlotJoined → True, PlotStyle → {RGBColor[1, 0, 0]}]
Show[{g3, g4}, PlotRange → {{0, 20}, {-1.69, -1.7}}]

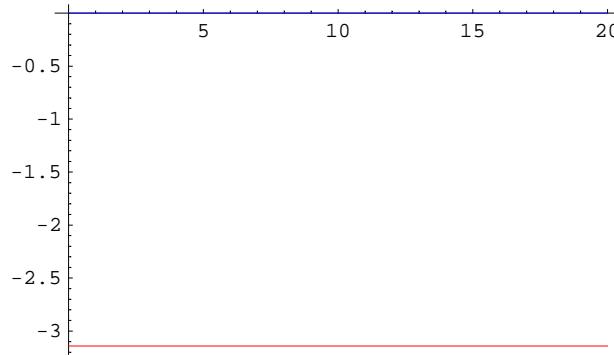
```



```

dat1 := Table[{e, 0}, {e, 0, 20}]
dat2 := Table[{f, -π}, {f, 0, 20}]
g5 := ListPlot[dat1, PlotJoined → True, PlotStyle → {RGBColor[0, 0, 1]}]
g6 := ListPlot[dat2, PlotJoined → True, PlotStyle → {RGBColor[1, 0, 0]}]
Show[{g5, g6}, PlotRange → All]

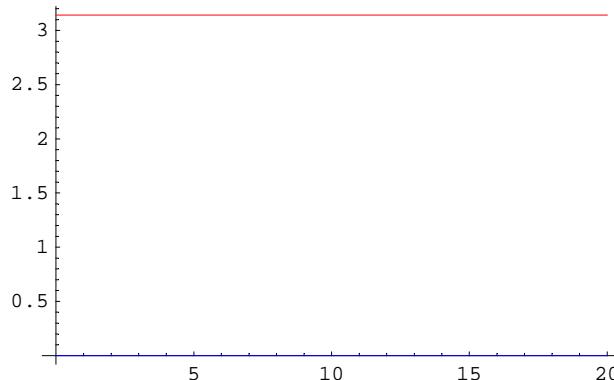
```



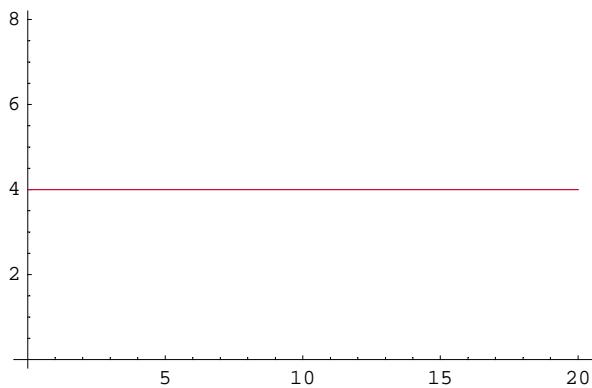
```

dat1 := Table[{g, 0}, {g, 0, 20}]
dat2 := Table[{h, π}, {h, 0, 20}]
g7 := ListPlot[dat1, PlotJoined → True, PlotStyle → {RGBColor[0, 0, 1]}]
g8 := ListPlot[dat2, PlotJoined → True, PlotStyle → {RGBColor[1, 0, 0]}]
Show[{g7, g8}, PlotRange → All]

```



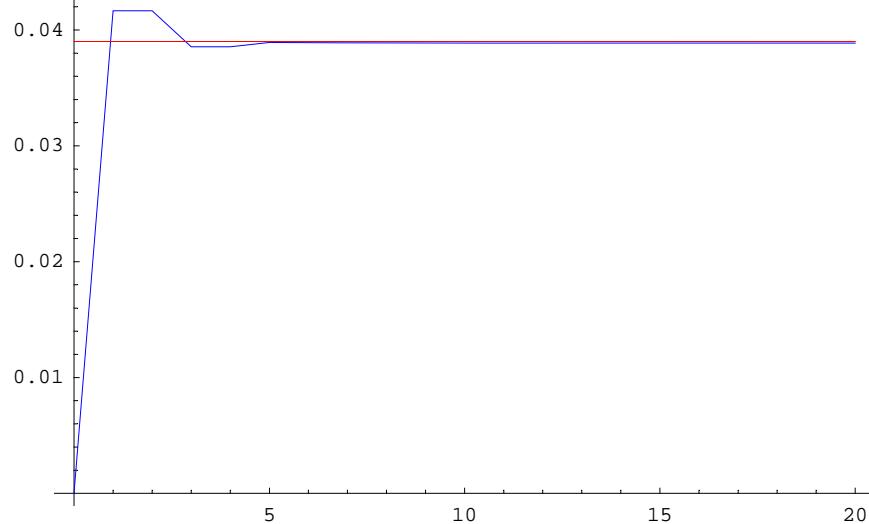
```
dat1 := Table[{i, 4}, {i, 0, 20}]
dat2 := Table[{j, 4}, {j, 0, 20}]
g9 := ListPlot[dat1, PlotJoined -> True, PlotStyle -> {RGBColor[0, 0, 1]}]
g10 := ListPlot[dat2, PlotJoined -> True, PlotStyle -> {RGBColor[1, 0, 0]}]
Show[{g9, g10}, PlotRange -> All]
```



```

dat1 := Table[{a, 2 \int_0^{0.5} \left( -\sum_{m=0}^a \frac{1}{m+1} (R)^{m+1} \cos \left[ \frac{(m+1) \pi}{2} \right] \right) dR}, {a, 0, 20}]
dat2 := Table[{b, 0.039}, {b, 0, 20}]
g1 := ListPlot[dat1, PlotJoined -> True, PlotStyle -> {RGBColor[0, 0, 1]}]
g2 := ListPlot[dat2, PlotJoined -> True, PlotStyle -> {RGBColor[1, 0, 0]}]
Show[{g1, g2}, PlotRange -> All]

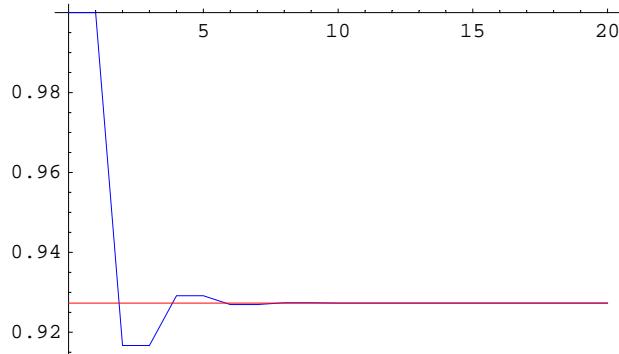
```



```

dat1 := Table[{a, 2 \int_0^{0.5} \left( \sum_{m=0}^a (R)^m \sin \left[ \frac{(m+1) \pi}{2} \right] \right) dR}, {a, 0, 20}]
dat2 := Table[{b, 0.927295}, {b, 0, 20}]
g1 := ListPlot[dat1, PlotJoined -> True, PlotStyle -> {RGBColor[0, 0, 1]}]
g2 := ListPlot[dat2, PlotJoined -> True, PlotStyle -> {RGBColor[1, 0, 0]}]
Show[{g1, g2}, PlotRange -> All]

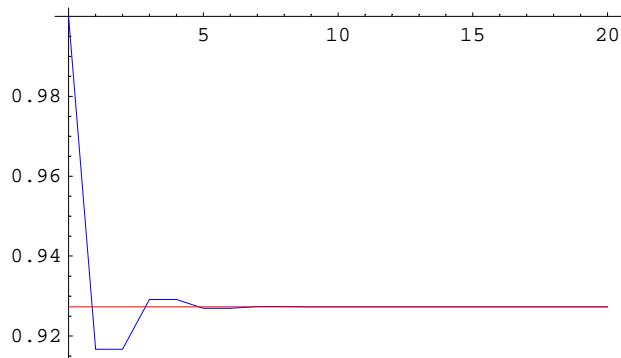
```



```

dat1 := Table[{\!m, 2 \!\!\! \int_0^{0.5} \left(1 + \sum_{m=0}^{\!a} (R)^{m+1} \cos\left[\frac{(m+1)\pi}{2}\right]\right) dR}, {a, 0, 20}]
dat2 := Table[{b, 0.927295}, {b, 0, 20}]
g1 := ListPlot[dat1, PlotJoined -> True, PlotStyle -> {RGBColor[0, 0, 1]}]
g2 := ListPlot[dat2, PlotJoined -> True, PlotStyle -> {RGBColor[1, 0, 0]}]
Show[{g1, g2}, PlotRange -> All]

```



```

dat1 := Table[{c, 2 \int_0^{0.5} \left( -\sum_{m=0}^c ((m+1) (R)^m \sin[\frac{(m+1)\pi}{2}]) \right) dR}, {c, 0, 20}]
dat2 := Table[{d, -0.8}, {d, 0, 20}]
g1 := ListPlot[dat1, PlotJoined → True, PlotStyle → {RGBColor[0, 0, 1]}]
g2 := ListPlot[dat2, PlotJoined → True, PlotStyle → {RGBColor[1, 0, 0]}]
Show[{g1, g2}, PlotRange → All]

```

