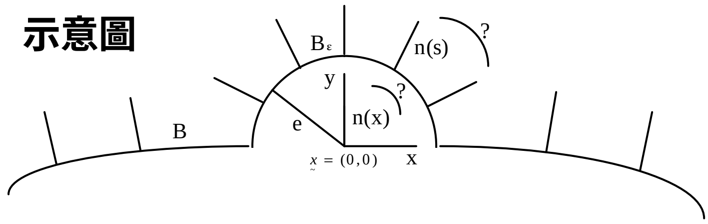
Derivation of dual BIE (including the singularity)

示意圖



Defined：

$$\underset{\sim}{x} = (0,0)\,,\ \underset{\sim}{s} = (e\cos q, e\sin q)$$

$$n(x) = \overline{n} = (0,1)\,,\ n(s) = n = (\cos q, \sin q)$$

$$y_i = x_i - s_i = (-e\cos q, -e\sin q)$$

$$u(s) = u(x) + \frac{\partial u(s)}{\partial s_1} e\cos q + \frac{\partial u(s)}{\partial s_2} e\sin q$$

$$t(s) = \frac{\partial u(s)}{\partial s_1}\cos q + \frac{\partial u(s)}{\partial s_2}\sin q$$

$$U(s,x) = \ln r = \ln e$$

$$\int_B U(s,x)t(s)dB(s) = e\ln e\int_0^p \left(\frac{\partial u(s)}{\partial s_1}\cos q + \frac{\partial u(s)}{\partial s_2}\sin q\right)\Big|_{s=x} dq = 0 \quad (e \to 0)$$

$$T(s,x) = -\frac{y_i n_i}{r^2} = \frac{1}{e}$$

$$\int_B T(s,x)u(s)dB(s) = \int_0^p \frac{1}{e}\left(u(x) + \frac{\partial u(s)}{\partial s_1}e\cos q + \frac{\partial u(s)}{\partial s_2}e\sin q\right)\Big|_{s=x} e\,dq = pu(x) \quad (e \to 0)$$

域內點邊界積分方程式

$$2pu(x) = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s)$$

$$2pu(x) = C.P.V.\int_B T(s,x)u(s)dB(s) + pu(x) - R.P.V.\int_B U(s,x)t(s)dB(s)$$

$$\Rightarrow pu(x) = C.P.V.\int_B T(s,x)u(s)dB(s) - C.P.V.\int_B U(s,x)t(s)dB(s)$$

$$L(s,x) = \frac{y_i \overline{n}_i}{r^2} = -\frac{\sin q}{e}$$

$$\int_B L(s,x)t(s)dB(s) = \int_0^p \left(-\frac{\sin q}{e}\right)\left(\frac{\partial u(s)}{\partial s_1}\cos q + \frac{\partial u(s)}{\partial s_2}\sin q\right)\Big|_{s=x} e\,dq = -\frac{p}{2}t(s)$$

$$M(s,x) = \frac{2y_i y_j n_i \overline{n}_j}{r^4} - \frac{n_i \overline{n}_i}{r^2} = \frac{\sin q}{e^2}$$

$$\int_B M(s,x)u(s)dB(s) = \int_0^p \frac{\sin q}{e^2}\left(u(x) + \frac{\partial u(s)}{\partial s_1}e\cos q + \frac{\partial u(s)}{\partial s_2}e\sin q\right)\Big|_{s=x} e\,dq = \frac{2}{e}u(x) + \frac{p}{2}t(s)$$

域內點邊界積分方程式

$$2pt(x) = \int_B M(s,x)u(s)dB(s) - \int_B L(s,x)t(s)dB(s)$$

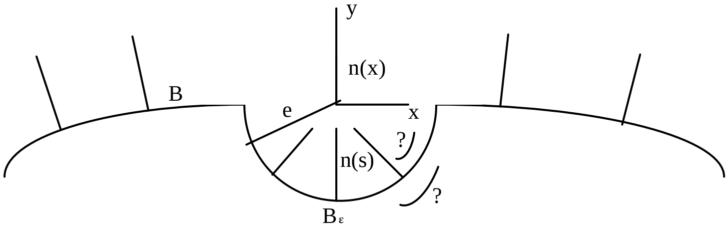
$$2pt(x) = H.P.V.\int_B M(s,x)u(s)dB(s) + \frac{p}{2}t(s) - C.P.V.\int_B L(s,x)t(s)dB(s) + \frac{p}{2}t(s)$$

$$\Rightarrow pt(x) = H.P.V.\int_B M(s,x)u(s)dB(s) - C.P.V.\int_B L(s,x)t(s)dB(s)$$

$$= C.P.V.\int_B M(s,x)u(s)dB(s) + \frac{2}{e}u(x) - C.P.V.\int_B L(s,x)t(s)dB(s)$$

Derivation of dual BIE (excluding the singularity)

示意圖



Defined：

$$\underset{\sim}{x} = (0,0)\ ,\ \underset{\sim}{s} = (e\cos q, -e\sin q)\ ,\ 0 < q < p$$

$$n(x) = \overline{n} = (0,1)\ ,\ n(s) = n = (-\cos q, \sin q)$$

$$y_i = x_i - s_i = (-e\cos q, e\sin q)$$

$$u(s) = u(x) + \frac{\partial u(s)}{\partial s_1} e\cos q - \frac{\partial u(s)}{\partial s_2} e\sin q$$

$$t(s) = -\frac{\partial u(s)}{\partial s_1}\cos q + \frac{\partial u(s)}{\partial s_2}\sin q$$

$$U(s,x) = \ln r = \ln e$$

$$\int_B U(s,x)t(s)dB(s) = e\ln e\int_0^p \left(-\frac{\partial u(s)}{\partial s_1}\cos q + \frac{\partial u(s)}{\partial s_2}\sin q\right)\Big|_{s=x} dq = 0 \quad (e \to 0)$$

$$T(s,x) = -\frac{y_i n_i}{r^2} = -\frac{1}{e}$$

$$\int_B T(s,x)u(s)dB(s) = \int_0^p -\frac{1}{e}\left(u(x) + \frac{\partial u(s)}{\partial s_1} e\cos q - \frac{\partial u(s)}{\partial s_2} e\sin q\right)\Big|_{s=x} e\,dq = -pu(x) \quad (e \to 0)$$

域外點邊界積分方程式

$$0 = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s)$$

$$0 = C.P.V.\int_B T(s,x)u(s)dB(s) - pu(x) - R.P.V.\int_B U(s,x)t(s)dB(s)$$

$$\Rightarrow pu(x) = C.P.V.\int_B T(s,x)u(s)dB(s) - C.P.V.\int_B U(s,x)t(s)dB(s)$$

$$L(s,x) = \frac{y_i \overline{n}_i}{r^2} = \frac{\sin q}{e}$$

$$\int_B L(s,x)t(s)dB(s) = \int_0^p \frac{\sin q}{e}\left(-\frac{\partial u(s)}{\partial s_1}\cos q + \frac{\partial u(s)}{\partial s_2}\sin q\right)\Big|_{s=x} e\,dq = \frac{p}{2}t(s)$$

$$M(s,x) = \frac{2y_i y_j n_i \overline{n}_j}{r^4} - \frac{n_i \overline{n}_i}{r^2} = \frac{\sin q}{e^2}$$

$$\int_B M(s,x)u(s)dB(s) = \int_0^p \frac{\sin q}{e^2}\left(u(x) + \frac{\partial u(s)}{\partial s_1} e\cos q - \frac{\partial u(s)}{\partial s_2} e\sin q\right)\Big|_{s=x} e\,dq = \frac{2}{e}u(x) - \frac{p}{2}t(s)$$

域外點邊界積分方程式

$$0 = \int_B M(s,x)u(s)dB(s) - \int_B L(s,x)t(s)dB(s)$$

$$0 = H.P.V.\int_B M(s,x)u(s)dB(s) - \frac{p}{2}t(s) - C.P.V.\int_B L(s,x)t(s)dB(s) - \frac{p}{2}t(s)$$

$$\Rightarrow pt(x) = H.P.V.\int_B M(s,x)u(s)dB(s) - C.P.V.\int_B L(s,x)t(s)dB(s)$$

$$= C.P.V.\int_B M(s,x)u(s)dB(s) + \frac{2}{e}u(x) - C.P.V.\int_B L(s,x)t(s)dB(s)$$

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