Some interesting properties of operators

①. $r^4 = 1$， $r = 1, -1, i, -i$

②. $\mathrm{FF}(f(t)) = 2\pi f(-t)$， $\mathrm{F}(f(t)) = 4\pi^2 f(t)$

where F is Fourier transform.

③. $\mathrm{HH}\ (y(t)) = -y(t)$

where H is the Hilbert transform.

④. $HH\ \underset{\sim}{y} = \underset{\sim}{y}$ ， $H^2 = I$

where H is Householder matrix.

⑤. $\mathrm{MM}\ (\cos m\theta) = -\pi^2 \dfrac{d^2}{d\theta^2}(\cos m\theta)$

where M is the integral operator of $M(s,x)$ kernel.

⑥. $\mathrm{UU}(\cos m\theta) = -\pi^2 \iint (\cos m\theta) d\phi d\theta$

where U is the integral operator of $U(s,x)$ kernel.

⑦. $\mathrm{T}^i \mathrm{T}^e = \mathrm{UM}$ ， $\mathrm{L}^i \mathrm{L}^e = \mathrm{MU}$

⑧. $i^2 = -1$

⑨. $C^3 = I$

where $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is a circulant matrix.

⑩. $I^2 = I$， $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

⑪. $J^2 = -J$， $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

⑫. $\mathrm{LL}\ (at^2 y''(t) + bty'(t) + cy(t)) = at^2 y''(t) + bty'(t) + cy(t)$

where L is the Laplace transform.

⑬. Abel transform $A(\varphi(z)) = \dfrac{1}{\sqrt{\pi}} \int_0^z \dfrac{\varphi(t)}{\sqrt{z-t}} dt$ then $DA^2(\varphi(z)) = \varphi(z)$