

### Three methods for boundary integrals of circle

|           | $\oint \frac{1}{1+p^2-2p\cos q} dq$ | $\oint \frac{\cos 2q}{1+p^2-2p\cos q} dq$ | $\oint \frac{1-p\cos q}{1+p^2-2p\cos q} dq$ | $\oint \frac{\sin q}{1+p^2-2p\cos q} dq$ | $\oint \ln \sqrt{(1+p^2-2p\cos q)} dq$ |
|-----------|-------------------------------------|---|---|--|--|
| $ p  < 1$ | $\frac{2p}{1-p^2}$                  | $\frac{2p p^2}{1-p^2}$                    | $2p$  | 0  | 0                                      |
| $ p  > 1$ | $\frac{2p}{p^2-1}$                  | $\frac{2p}{p^4-p^2}$                      | 0   | 0  | $2p \ln p$                             |

Method 1 : Residue theory  $\cos q = \frac{1}{2}(z + z^{-1})$ ,  $\sin q = \frac{1}{2i}(z - z^{-1})$

Method 2 : Degenerate kernel in potential theory  $(U, T, L, M)$

Method 3 : Poisson integral formula  $u(\mathbf{r}, \mathbf{f}) = \frac{1}{2\pi} \int_0^{2\pi} \frac{(R^2 - r^2) f(q)}{R^2 + r^2 - 2Rr \cos(q - f)} dq$