Wanted solve system	Fundamental solution
$\frac{d^2u(x)}{dx^2}=0$， $0<x<\ell$， $u(0)=a$， $u(\ell)=b$	$\frac{d^2U(x,s)}{dx^2}=\delta(x-s)$ $U(x,s)=\frac{1}{2}\lvert x-s\rvert$

$$\int_0^\ell \left(v\frac{d^2u}{dx^2}-u\frac{d^2v}{dx^2}\right)dx=\left(v\frac{du}{dx}-u\frac{dv}{dx}\right)\Bigg|_{x=0}^{x=\ell}$$

$$u(s)=\left(u(x)\frac{dU(x,s)}{dx}-U(x,s)\frac{du(x)}{dx}\right)\Bigg|_{x=0}^{x=\ell},\quad 0<s<\ell$$

$$u(x)=\left(u(s)\frac{dU(s,x)}{ds}-U(s,x)\frac{du(s)}{ds}\right)\Bigg|_{s=0}^{s=\ell}=\left[u(s)T(s,x)-U(s,x)u'(s)\right]\Big|_{s=0}^{s=\ell},\quad 0<x<\ell$$

$$U(s,x)=\begin{cases}\frac{1}{2}(s-x), & s>x\\ \frac{1}{2}(x-s), & x>s\end{cases}\quad T(s,x)=\begin{cases}\frac{1}{2}, & s>x\\ -\frac{1}{2}, & x>s\end{cases}\quad L(s,x)=\begin{cases}-\frac{1}{2}, & s>x\\ \frac{1}{2}, & x>s\end{cases}\quad M(s,x)=\begin{cases}0, & s>x\\ 0, & x>s\end{cases}$$

$x\to 0^-$,

$$u(0^-)=T(\ell,0^-)u(\ell)-U(\ell,0^-)u'(\ell)-T(0,0^-)u(0)+U(0,0^-)u'(0)=\frac{b}{2}-\frac{\ell}{2}u'(\ell)-\frac{a}{2}=0$$

$x\to \ell^+$,

$$u(\ell^+)=T(\ell,\ell^+)u(\ell)-U(\ell,\ell^+)u'(\ell)-T(0,\ell^+)u(0)+U(0,\ell^+)u'(0)=-\frac{b}{2}+\frac{a}{2}+\frac{\ell}{2}u'(0)=0$$

$$u'(\ell)=\frac{b-a}{\ell},\quad u'(0)=\frac{b-a}{\ell}$$

$$u(x)=T(\ell,x)u(\ell)-U(\ell,x)u'(\ell)-T(0,x)u(0)+U(0,x)u'(0)=\frac{(b-a)x}{\ell}+a$$ **Correct!**

$$u'(x)=\left[u(s)M(s,x)-L(s,x)t(s)\right]\Big|_{s=0}^{s=\ell},\quad 0<x<\ell$$

$$x\to 0^-,\quad u'(0^-)=-L(\ell,0^-)u'(\ell)+L(0,0^-)u'(0)=\frac{1}{2}u'(\ell)-\frac{1}{2}u'(0)=0$$

$$x\to \ell^+,\quad u'(\ell^+)=-L(\ell,\ell^+)u'(\ell)+L(0,\ell^+)u'(0)=-\frac{1}{2}u'(\ell)+\frac{1}{2}u'(0)=0$$

Dependent equation

BEM 2006 HW 5 BY TASHI