go about it, a direct measure of $B_n = D_s$ is clearly feasible. While it may be true that, for some purposes, it can be reasonable to concentrate one's attention upon the total field, it is seen from the above that numerical studies of B_n are neither "vacuous" nor "misleading."

Whether or not one regards the resonant scattering magnitudes as "extraordinarily large" seems to be a matter of semantics. The full-space cases displayed in Ref. 3 yield B_n/A magnitudes in the range of 0.5–10. We do not regard such figures as "extraordinarily large." One the other hand, the magnitude with respect to the free field is, of course, $B_n/ka \simeq 10^2 \times B_n$ near resonance, which is indeed quite large, but in this case Twersky seems to be chiefly marveling at the well-known "giant monopole" resonance of the single bubble.[5,6]

One would like to devise the optimum experimental setup to observe these multiple scatter resonances. It would certainly be *convenient* to measure the total scattered field $\mathcal{G}$ for some symmetrical array of resonators. But in the *full-space case*, this implies some form of broadside incidence—for which the resonance effect disappears for the cases examined (Figs. 4, 5, 7, 8 and of Ref. 3). But when a plane array is backed by a thin plate (a membrane under tension would also do), the resonance (a *bona fide* pole or *superresonance* in this case) *is* present for broadside incidence (Figs. 14–17), and a completely symmetric experiment is possible (an even simpler model would consist of a doublet in a narrow-bore perfect cylindrical waveguide, with a compact source in the normal plane bisecting the doublet).

As a final remark, I must insist that in all our published discussions of these effects and in Ref. 3 in particular, we have dealt only with the very simplest models, so as to display the phenomena in their most obvious forms. We have tried to make our results as clear as possible by keeping unnecessary mathematical developments at a minimum and by providing numerous figures—out of consideration for our readers.

[1]V. Twersky, "Multiple scattering by finite regular arrays of resonators," J. Acoust. Soc. Am. 87, 25–41 (1990).

[2]I. Tolstoy, "Superresonant systems of scatters. I," J. Acoust. Soc. Am. 80, 282–294 (1986); 81, 1987 (1987).

[3]I. Tolstoy and A. Tolstoy, "Superresonant systems of scatterers. II," J. Acoust. Soc. Am. 83, 2086–2096 (1988).

[4]G. Gaunaurd, K. P. Scharnhorst, and H. Uberall, "Giant monopole resonances," J. Acoust. Soc. Am. 65, 573–594 (1979).

[5]A. P. Dowling and J. E. Ffowcs-Williams, *Sound and Sources of Sound* (Harwood, London, 1983).

[6]M. J. Lighthill, *Waves in Fluids* (Cambridge U. P., Cambridge, 1978).

Comments on resonant systems of scatterers

Victor Twersky

Mathematics Department, University of Illinois, Chicago, Illinois 60680

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Details are provided for aspects of scattering by resonant systems.

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This letter provides contexts and details for remarks quoted[1] from a recent article[2] on multiple scattering by finite regular arrays of resonators. Some of the remarks apply for all distances of observation, and the rest hold for the far field.

The article[2] analyzes scattering of an excess pressure field $\phi = \exp(i\mathbf{k}\cdot\mathbf{r})$ by arrays of monopoles (of radius a) with centers at $\mathbf{b}_s = b\hat{\mathbf{b}}_s$ around the origin $r = 0$, for arbitrary directions of incidence $(\hat{\mathbf{k}})$ and observation $(\hat{\mathbf{r}})$. Applying earlier results,[3] the scattered field $\mathcal{U}$ is given in terms of the appropriate explicit coefficients $D_s(\hat{\mathbf{k}})$ for seven different regular arrays (with minimum separation d of neighbors) for all values of $r>0$ external to the obstacles. As kd increases, D_s reduces to the isolated monopole scattering coefficient a_0. For $r<b$, $\mathcal{U}$ consists of standing waves $j_n(kr)$; for $r>b$, $\mathcal{U}$ consists of radiating waves $h_n^{(1)}(kr)$. The internal field of an individual obstacle(s) follows from continuity of the total field $\Psi(\mathbf{r}) = \phi + \mathcal{U}$ evaluated at $\mathbf{r} = \mathbf{b}_s + \mathbf{a}$.

If $ka \to 0$ for any value of r, then $\mathcal{U} \to 0$ and $\Psi \to \phi$. There are no singularities in $\mathcal{U}$. Discussions[4] of "real poles" and of the "removal of infinities" by the introduction of radiation damping and nonzero radii are vacuous.

Discussions[5] and plots of Figs. 3–11 for an individual D_s are misleading. Key features are distorted because "peaks narrower than $2ka$... have been truncated at width $2ka$" to display $|D_s/a_0| \equiv M$. Thus, for axial incidence on the doublet (Fig. 3), at coordinate values $(ka,kd) = (0.01389, 0.55)$, the "maximum effective" peak is given as $M \simeq 7$ instead of the actual $M \simeq 10$; this value is not the largest in the range shown for ka, i.e., $M \simeq 24$ at $(0.0140, 0.3553)$. (The range could be extended to pick up an additional order of magnitude and still maintain the restriction that d be sufficiently larger than $2a$ for the simple monopole development to apply[2,3].) The discussions[5] obscure the essential physics. The physical interpretation of $D_s(\hat{\mathbf{k}})$ for an array with n different separations $|\mathbf{b}_s - \mathbf{b}_t| \geqslant d$ follows directly from its decomposition[2] in terms of $n+1$ $\hat{\mathbf{k}}$-independent oscillator mode coefficients: All characteristics of $D_s(\hat{\mathbf{k}})$ are determined by coupling of the $n+1$ collective oscillators that represent the array. For example, the doublet is represented by coupled mode-0 and mode-1 oscillators; at axial incidence, the peaks correspond to mode-1 resonances detuned slightly by coupling with mode 0.

The discussion of an "obstacle/barrier" half-plane[5] for Fig. 12 is misleading; the half-plane perpendicular to the triangular array (with edge at its geometrical center) gives rise to a more complicated four-obstacle problem than indicated. At a simplified (and incomplete) level, the incident wave and the three resonators excite cylindrical waves radiated by the edge of the half-plane, and the two flanking resonators excite reflected as well as transmitted waves.

If measurements of $\mathscr{U}$ are feasible in the near field $\mathbf{r} \approx \mathbf{b}_s + a$ of obstacle(s) under conditions for which the fields of all neighbors are negligible, then a coefficient $D_s(\hat{\mathbf{k}})$ could constitute an observable. However, such measurements are not possible in the far field of the array ($r \gg b$), the context[2] for the remaining quotations.[1] In the far field, r/b and kr large, $\mathscr{U}$ factors[3] to $h_0^{(1)}(kr)\mathscr{G}(\hat{\mathbf{r}},\hat{\mathbf{k}})$ where the scattering amplitude $\mathscr{G}$ is basic to applications. The scattering cross section $S(\hat{\mathbf{k}})$ obtained from $\mathscr{G}$ determines the net energy outflow from the system. All values of $|\mathscr{G}(\hat{\mathbf{k}},\hat{\mathbf{k}})|$ and $S(\hat{\mathbf{k}})$ are less[2] than twice the maximal values of the single scattering approximations.

The only observable scattering amplitude for the system of resonators in a medium free of other obstacles is $\mathscr{G}(\hat{\mathbf{r}},\hat{\mathbf{k}})$. An individual D_s is not observable via a scattering amplitude unless $D_s = D$, the special cases of symmetrically excited planar arrays for which $\mathscr{G}$ is proportional to D. Numerical computations[5] for an individual D_s do not represent physically observable far-field data, and their peaks and locations (ka,kd) are not representative of the values for maximal scattering by the system as a whole. In particular, for polygonal arrays, the maximum values of $|\mathscr{G}|$ and S occur for broadside incidence.[2]

[1] I. Tolstoy, J. Acoust. Soc. Am. **88**, 1178–1179 (1990).
[2] V. Twersky, J. Acoust. Soc. Am. **87**, 25 (1990).
[3] V. Twersky, J. Math. Phys. **3**, 88 (1962); J. Opt. Soc. Am. **52**, 145 (1962).
[4] I. Tolstoy, J. Acoust. Soc. Am. **80**, 282 (1986); **81**, 1987 (1987).
[5] I. Tolstoy and A. Tolstoy, J. Acoust. Soc. Am. **83**, 2086 (1988).

ERRATUM

Erratum: "The *k*-space formulation of the acoustic scattering problem" [J. Acoust. Soc. Am. 84, 1890–1893 (1988)]

Shozo Koshigoe
Code 3892, Research Department, Naval Weapons Center, China Lake, California 93555

Norbert N. Bojarski
1320 Santiago Drive, Newport Beach, California 92660-4944

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The second-named author has requested that an erratum note be published stating that he is not a co-author of this paper.

Daniel W. Martin
Editor-in-Chief