On the screw dislocation in a functionally graded piezoelectric plane and half-plane

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Abstract

In this paper closed-form expressions of the electroelastic field induced by a piezoelectric screw dislocation in a functionally graded piezoelectric plane and half-plane are derived. The material properties are assumed to vary exponentially along the x and y-directions. The solution for a screw dislocation in a functionally graded piezoelectric plane is obtained through introduction of two generalized stress functions. The solution for a screw dislocation in a functionally graded piezoelectric half-plane is derived by using the method of image. It is also found that the interaction between a piezoelectric screw dislocation and a circular insulating hole in the functionally graded piezoelectric material can be solved by using series expansion method.

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Keywords: Functionally graded piezoelectric material; Screw dislocation; Stress function; Method of image; Half-plane; Circular hole

1. Introduction

Functionally graded piezoelectric materials (FGPMs) usually refer to the pieozoelectric materials in which the material properties vary smoothly in a given direction. FGPMs can be utilized to provide the desirable thermal-mechanical and piezoelectric properties. Applications of FGPMs can be found in tribology, electronic and biomechanics. Recent analysis (Almajid and Taya, 2001; Wu et al., 2002; Taya et al., 2003) and experimental tests (Zhu et al., 2000; Almajid and Taya, 2001; Taya et al., 2003) have clearly shown the advantage of FGPM actuator or sensor over the traditional ones. Due to their potential application in intelligent/smart structures, FGPMs have attracted wide attention in recent years (see for example Jin and Zhong, 2002; Li and Weng, 2002; Pan and Han, 2005; Collet et al., 2006). A review of the principal developments in functionally graded materials (FGMs) since 2000 can be found in the most recent work by Birman and Byrd (2007).

Most recently by means of the stress function technique, Lazar (2007) addressed a screw dislocation in an elastic medium exponentially graded in the y-direction within the framework of the classical elasticity theory.

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31 Motivated by Lazar (2007), we investigate in this paper a piezoelectric screw dislocation in an FGPM plane
32 and half-plane within the framework of linear theory of piezoelectricity. Also discussed is a piezoelectric screw
33 dislocation near a circular insulating hole in an FGPM plane.

34 ## 2. A piezoelectric screw dislocation in an FGPM plane

35 We first consider a static screw dislocation located at origin in an FGPM plane. The FGPM plane is trans-
36 versely isotropic with the poling direction parallel to the z-axis. The screw dislocation is assumed to be straight
37 and infinitely long in the z-direction, suffering a displacement jump b and an electric potential jump $\Delta\phi$ across
38 the slip plane.
39 The engineering shear strains γ_{zx}, γ_{zy} and the electric fields E_x, E_y should satisfy the following incompatibility
40 conditions (Lazar, 2007):
41

$$\frac{\partial \gamma_{zy}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} = b\delta(x)\delta(y), \tag{1}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\Delta\phi\delta(x)\delta(y), \tag{2}$$

43

44 where $\delta()$ is the Dirac delta function.
45 In addition the strains and electric fields can be expressed in terms of stresses σ_{zx}, σ_{zy} and electric displace-
46 ments D_x, D_y as
47

$$\begin{bmatrix} \gamma_{zy} \\ E_y \end{bmatrix} = \frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} \sigma_{zy} \\ D_y \end{bmatrix}, \tag{3a}$$

$$\begin{bmatrix} \gamma_{zx} \\ E_x \end{bmatrix} = \frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} \sigma_{zx} \\ D_x \end{bmatrix}, \tag{3b}$$

49

50 where c_{44}, e_{15} and ϵ_{11} are, respectively, the elastic modulus, the piezoelectric coefficient, and the dielectric per-
51 mittivity; $\tilde{c}_{44} = c_{44} + e_{15}^2/\epsilon_{11} \geqslant c_{44}$ is the piezoelectrically stiffened elastic coefficient. We remark that different
52 from the homogeneous material case, c_{44}, e_{15} and ϵ_{11} (and as a result $\tilde{c}_{44}$) in this paper are all functions of the
53 coordinates x and y. In particular, we assume that
54

56 $$c_{44} = \exp(2\beta_1 x + 2\beta_2 y)c_{44}^0, \quad e_{15} = \exp(2\beta_1 x + 2\beta_2 y)e_{15}^0, \quad \epsilon_{11} = \exp(2\beta_1 x + 2\beta_2 y)\epsilon_{11}^0, \tag{4}$$

57 where c_{44}^0, e_{15}^0, ϵ_{11}^0, β_1 and β_2 are material constants. It is noticed that the above assumption of exponential
58 variation for the material properties has been adopted by many authors (see for example, Jin and Zhong,
59 2002; Wang, 2003; Kwon, 2003; Collet et al., 2006) to simplify the analysis involved. Here it should be pointed
60 out that there exist explicit solutions for FGPMs with variations other than the exponential variations as-
61 sumed in this research (see for example, Collet et al., 2006).
62 Due to the fact that the stresses and electric displacements satisfy the following equilibrium equations:

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} = 0, \tag{5}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0, \tag{6}$$

64

65 then it's convenient to introduce the following generalized stress functions f and g which are related to the
66 stresses and electric displacements through
67

$$\sigma_{zy} = \frac{\partial f}{\partial x}, \quad \sigma_{zx} = -\frac{\partial f}{\partial y},$$

$$D_y = \frac{\partial g}{\partial x}, \quad D_x = -\frac{\partial g}{\partial y}. \tag{7}$$

69

70 As a result the strains and electric fields can be expressed in terms of the two introduced generalized stress
71 functions f and g as

$$\begin{bmatrix} \gamma_{zy} \\ E_y \end{bmatrix} = \frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} f_x \\ g_x \end{bmatrix}, \tag{8a}$$

$$\begin{bmatrix} \gamma_{zx} \\ E_x \end{bmatrix} = -\frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} f_y \\ g_y \end{bmatrix}. \tag{8b}$$

Inserting (8a) and (8b) into Eqs. (1) and (2) yields

$$\nabla^2 f - 2\beta_1 f_x - 2\beta_2 f_y = (c_{44}^0 b + e_{15}^0 \Delta\phi)\delta(x)\delta(y),$$
$$\nabla^2 g - 2\beta_1 g_x - 2\beta_2 g_y = (e_{15}^0 b - \epsilon_{11}^0 \Delta\phi)\delta(x)\delta(y), \tag{9}$$

We now introduce two new functions φ and ψ which are related to f and g through

$$f = \exp(\beta_1 x + \beta_2 y)\varphi, \quad g = \exp(\beta_1 x + \beta_2 y)\psi, \tag{10}$$

As a result Eq. (9) can be rewritten in terms of φ and ψ as

$$\nabla^2 \varphi - \beta^2 \varphi = (c_{44}^0 b + e_{15}^0 \Delta\phi)\delta(x)\delta(y),$$
$$\nabla^2 \psi - \beta^2 \psi = (e_{15}^0 b - \epsilon_{11}^0 \Delta\phi)\delta(x)\delta(y), \tag{11}$$

where $\beta = \sqrt{\beta_1^2 + \beta_2^2}$. The solutions to the two inhomogeneous Helmholtz equations in Eq. (11) can be expediently given by (Lazar, 2007)

$$\varphi = -\frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} K_0(\beta r),$$
$$\psi = \frac{\epsilon_{11}^0 \Delta\phi - e_{15}^0 b}{2\pi} K_0(\beta r), \tag{12}$$

where $r = \sqrt{x^2 + y^2}$ and K_n is the nth order modified Bessel function of the second kind.

In view of Eqs. (10) and (12), the explicit expressions of f and g are given by

$$f = -\frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} e^{\beta_1 x + \beta_2 y} K_0(\beta r),$$
$$g = \frac{\epsilon_{11}^0 \Delta\phi - e_{15}^0 b}{2\pi} e^{\beta_1 x + \beta_2 y} K_0(\beta r). \tag{13}$$

Substituting Eq. (13) into Eq. (7), we arrive at the expressions of stresses and electric displacements as

$$\sigma_{zx} = \frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} e^{\beta_1 x + \beta_2 y} \left[\beta_2 K_0(\beta r) - \beta\frac{y}{r} K_1(\beta r)\right],$$
$$\sigma_{zy} = \frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} e^{\beta_1 x + \beta_2 y} \left[\beta\frac{x}{r} K_1(\beta r) - \beta_1 K_0(\beta r)\right],$$
$$D_x = \frac{e_{15}^0 b - \epsilon_{11}^0 \Delta\phi}{2\pi} e^{\beta_1 x + \beta_2 y} \left[\beta_2 K_0(\beta r) - \beta\frac{y}{r} K_1(\beta r)\right], \tag{14}$$
$$D_y = \frac{e_{15}^0 b - \epsilon_{11}^0 \Delta\phi}{2\pi} e^{\beta_1 x + \beta_2 y} \left[\beta\frac{x}{r} K_1(\beta r) - \beta_1 K_0(\beta r)\right]. \tag{15}$$

Using Eqs. (3a) and (3b), the strains and electric fields can be derived to be

$$\gamma_{zx} = \frac{b}{2\pi} e^{-\beta_1 x - \beta_2 y} \left[\beta_2 K_0(\beta r) - \beta\frac{y}{r} K_1(\beta r)\right],$$
$$\gamma_{zy} = \frac{b}{2\pi} e^{-\beta_1 x - \beta_2 y} \left[\beta\frac{x}{r} K_1(\beta r) - \beta_1 K_0(\beta r)\right],$$
$$E_x = \frac{\Delta\phi}{2\pi} e^{-\beta_1 x - \beta_2 y} \left[\beta\frac{y}{r} K_1(\beta r) - \beta_2 K_0(\beta r)\right], \tag{16}$$
$$E_y = \frac{\Delta\phi}{2\pi} e^{-\beta_1 x - \beta_2 y} \left[\beta_1 K_0(\beta r) - \beta\frac{x}{r} K_1(\beta r)\right]. \tag{17}$$

It is observed from Eqs. (16) and (17) that the displacement jump b can only induce the mechanical strains γ_{zx}, γ_{zy}; whilst the electric potential jump $\Delta\phi$ can only induce the electric fields E_x, E_y. In addition the strains and electric fields are independent of the specific values of the three material constants c_{44}^0, e_{15}^0 and ϵ_{11}^0.

Furthermore the stresses and electric displacements can be expressed in the polar coordinate system as

$$\sigma_{z\theta} = \frac{\partial f}{\partial r} = \frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} e^{r(\beta_1 \cos\theta + \beta_2 \sin\theta)} [\beta K_1(\beta r) - (\beta_1 \cos\theta + \beta_2 \sin\theta) K_0(\beta r)],$$

$$\sigma_{zr} = -\frac{\partial f}{r\partial\theta} = \frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} (\beta_2 \cos\theta - \beta_1 \sin\theta) e^{r(\beta_1 \cos\theta + \beta_2 \sin\theta)} K_0(\beta r), \tag{18}$$

$$D_\theta = \frac{\partial g}{\partial r} = \frac{e_{15}^0 b - \epsilon_{11}^0 \Delta\phi}{2\pi} e^{r(\beta_1 \cos\theta + \beta_2 \sin\theta)} [\beta K_1(\beta r) - (\beta_1 \cos\theta + \beta_2 \sin\theta) K_0(\beta r)],$$

$$D_r = -\frac{\partial g}{r\partial\theta} = \frac{e_{15}^0 b - \epsilon_{11}^0 \Delta\phi}{2\pi} (\beta_2 \cos\theta - \beta_1 \sin\theta) e^{r(\beta_1 \cos\theta + \beta_2 \sin\theta)} K_0(\beta r). \tag{19}$$

It can be easily checked that when we ignore the piezoelectric effect, i.e., $e_{15} = 0$ and let the material be graded only in the y-direction, i.e., $\beta_1 = 0$, Eqs. (14), (16) and (19) reduce to those derived by Lazar (2007). On the other hand if we ignore the material gradation, i.e., $\beta_1 = \beta_2 = 0$, Eqs. (14)–(19) will reduce to those derived by Pak (1990) (also see Lee et al., 2000) by observing the following asymptotic behaviors for $K_0(x)$ and $K_1(x)$

$$K_0(x) \to -\ln(x/2), K_1(x) \to x^{-1}, \quad \text{when } x \to 0^+ \tag{20}$$

We point out that with the derived full-field expressions for the stresses and electric displacements, the generalized Peach-Koehler force acting on the piezoelectric screw dislocation can be obtained by using the Peach-Koehler formulation (Lee et al., 2000). As expected the Peach-Koehler force is no longer a central force due to the existence of the tangential component of the force in the polar coordinates (Lazar, 2007).

3. A piezoelectric screw dislocation in an FGPM half-plane

3.1. An FGPM half-plane with a traction-free and charge-free surface

We now consider the case where the piezoelectric screw dislocation is located at $(0, d)$, $(d > 0)$ in an FGPM half-plane $y \geqslant 0$. The material properties of the FGPM half-plane are also assumed to vary exponentially as described by Eq. (4). The boundary conditions on the surface of the FGPM half-plane is assumed to be traction-free and charge-free, i.e., $\sigma_{zy} = D_y = 0$ at $y = 0$. By using the image method (Pak, 1990), the two generalized stress functions f and g for a half-plane with a traction-free and charge-free surface are found to be

$$f = -\frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} e^{\beta_1 x + \beta_2(y+d)} \left\{ K_0[\beta\sqrt{x^2 + (y-d)^2}] - K_0[\beta\sqrt{x^2 + (y+d)^2}] \right\},$$

$$g = \frac{\epsilon_{11}^0 \Delta\phi - e_{15}^0 b}{2\pi} e^{\beta_1 x + \beta_2(y+d)} \left\{ K_0[\beta\sqrt{x^2 + (y-d)^2}] - K_0[\beta\sqrt{x^2 + (y+d)^2}] \right\}. \tag{21}$$

It can be easily checked that Eq. (21) satisfies the traction-free and charge-free boundary conditions $\sigma_{zy} = D_y = 0$ (or equivalently $f = g = 0$) on the surface $y=0$. Eq. (21) can be physically interpreted as the summation of the electroelastic field caused by a piezoelectric screw dislocation with displacement jump b and electric potential jump $\Delta\phi$ located at $(0, d)$ and another one by an *image* piezoelectric screw dislocation with displacement jump $*e^{2\beta_2 d} b$ and electric potential jump $-e^{2\beta_2 d}\Delta\phi$ located at the image point $(0, -d)$. It is noticed that the magnitudes of the displacement and electric potential jumps for the image screw dislocation at $(0, -d)$ are influenced by the gradient parameter $\beta_2 d$. Therefore with the derived generalized stress functions f and g, we can easily find all the electroelastic quantities. One special case is for the one when the material is graded only in one coordinate. For example, if we ignore the gradation in the x-direction, i.e., $\beta_1 = 0$, the stress component σ_{zy} and the electric displacement D_y can be expressed as

$$\sigma_{zy} = \frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi d} \Omega(x, y, \beta_2),$$

$$D_y = \frac{e_{15}^0 b - \epsilon_{11}^0 \Delta\phi}{2\pi d} \Omega(x, y, \beta_2),$$

(22)

where the function $\Omega(x, y, \beta_2)$ is defined by

$$\Omega(x, y, \beta_2) = d|\beta_2|e^{\beta_2(y+d)}x\left\{\frac{K_1[|\beta_2|\sqrt{x^2 + (y-d)^2}]}{\sqrt{x^2 + (y-d)^2}} - \frac{K_1[|\beta_2|\sqrt{x^2 + (y+d)^2}]}{\sqrt{x^2 + (y+d)^2}}\right\}.$$

(23)

It can also be easily checked that the results derived in this section can be reduced to those derived by Pak (1990) for a screw dislocation interacting with a traction-free and charge-free surface if we ignore the material gradation, i.e., $\beta_1 = \beta_2 = 0$. We present in Fig. 1 the contour plots of the function $\Omega(x, y, \beta_2)$ in Eq. (23) for $\beta_2 d = -3, -1, 1, 3$. It is observed that the gradient parameter β_2 can significantly influence the values of the function $\Omega(x,y,\beta_2)$, and consequently influence the stress component σ_{zy} and the electric displacement D_y.
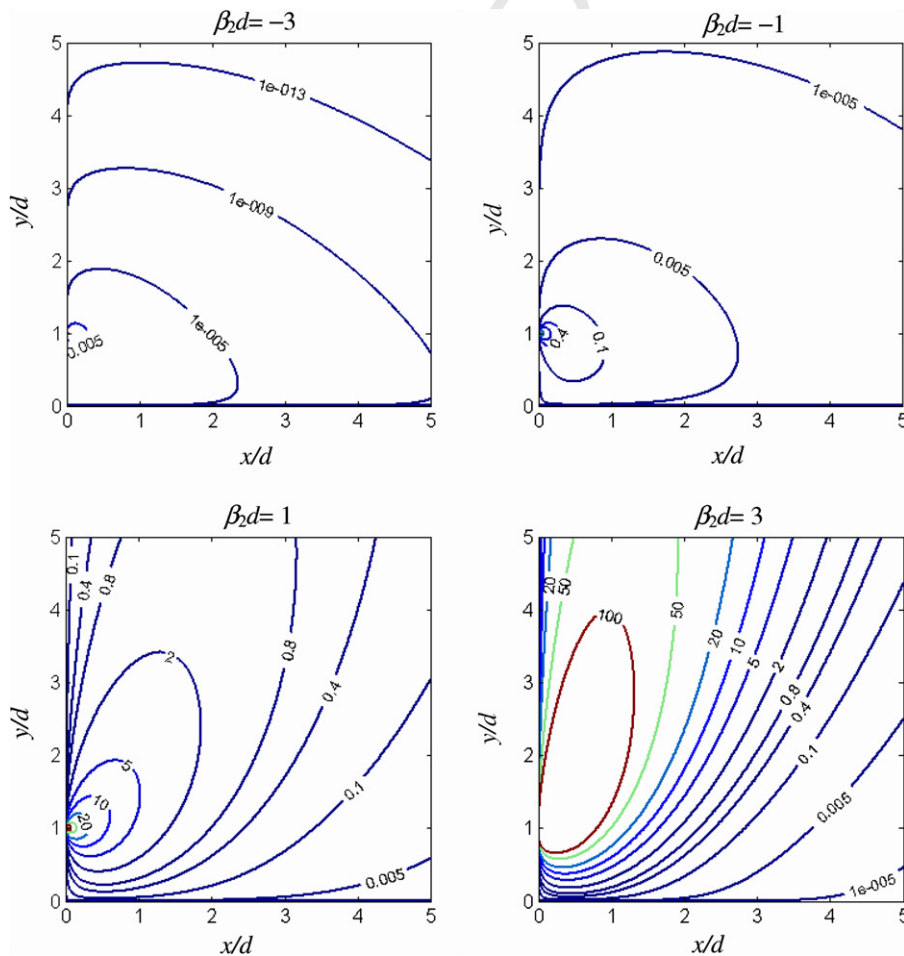

Fig. 1. Contours of the function $\Omega(x, y, \beta_2)$ in Eq. (23) for $\beta_2 d = -3, -1, 1, 3$.

157 ### 3.2. An FGPM half-plane bonded by a rigid conductor

158   It shall be mentioned that in general the image method cannot be easily applied to address the problem of a
159 piezoelectric screw dislocation located at $(0, d)$, $(d > 0)$ in an FGPM half-plane $y \geqslant 0$ bonded by a rigid con-
160 ductor, i.e., $\gamma_{zx} = E_x = 0$ on $y = 0$. However when we ignore the gradation in the y-direction, i.e., $\beta_2 = 0$ in Eq.
161 (4), the two generalized stress functions f and g for a half-plane bonded by a rigid conductor can still be easily
162 obtained by using the image method as follows

$$f = -\frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} e^{\beta_1 x} \left\{ K_0[|\beta_1|\sqrt{x^2 + (y-d)^2}] + K_0\left[|\beta_1|\sqrt{x^2 + (y+d)^2}\right] \right\},$$

$$g = \frac{\epsilon_{11}^0 \Delta\phi - e_{15}^0 b}{2\pi} e^{\beta_1 x} \left\{ K_0\left[|\beta_1|\sqrt{x^2 + (y-d)^2}\right] + K_0\left[|\beta_1|\sqrt{x^2 + (y+d)^2}\right] \right\},$$

(24)

164

165 which can be interpreted as the summation of the electroelastic field caused by a piezoelectric screw dislocation
166 with displacement jump b and electric potential jump $\Delta\phi$ located at $(0, d)$ and another one by an *image* pie-
167 zoelectric screw dislocation with displacement jump b and electric potential jump $\Delta\phi$ located at the image
168 point $(0, -d)$. In this case the full field expressions of γ_{zx} and E_x are given by

$$\gamma_{zx} = -\frac{|\beta_1| b}{2\pi} e^{-\beta_1 x} \left\{ \frac{(y-d)K_1[|\beta_1|\sqrt{x^2 + (y-d)^2}]}{\sqrt{x^2 + (y-d)^2}} + \frac{(y+d)K_1\left[|\beta_1|\sqrt{x^2 + (y+d)^2}\right]}{\sqrt{x^2 + (y+d)^2}} \right\},$$

$$E_x = \frac{|\beta_1|\Delta\phi}{2\pi} e^{-\beta_1 x} \left\{ \frac{(y-d)K_1\left[|\beta_1|\sqrt{x^2 + (y-d)^2}\right]}{\sqrt{x^2 + (y-d)^2}} + \frac{(y+d)K_1\left[|\beta_1|\sqrt{x^2 + (y+d)^2}\right]}{\sqrt{x^2 + (y+d)^2}} \right\},$$

(25)

170

171 which satisfy the boundary conditions $\gamma_{zx} = E_x = 0$ on $y = 0$.

172 ## 4. Conclusion and discussion

173   In this paper we derived closed-form solutions for a straight piezoelectric screw dislocation with a displace-
174 ment jump b and an electric potential jump $\Delta\phi$ in an FGPM plane by using the stress function technique and
175 in an FGPM half-plane by using the stress the method of image. The solutions were verified by comparison
176 with existing solutions (Pak, 1990; Lazar, 2007). It's expected that the derived solutions for a screw dislocation
177 can be further applied to investigate crack problems in FGPM. In fact by using the general solution to the
178 Helmholtz equations in Eq. (11) in polar coordinates, we can discuss more complex interaction problems.
179 For example, as shown in Fig. 2, we consider the case in which there is a circular insulating (or charge-free)
180 hole of radius R in an FGPM plane described by Eq. (4). The center of the circular hole is at the origin of the
181 coordinate system, and the piezoelectric screw dislocation is located at $(\delta, 0)$, $(\delta > R)$ on the positive real axis
182 in the matrix. The two generalized stress functions f and g for this case can be expressed in infinite series form
183 as
184

$$f = -\frac{c_{44}^0 b + e_{15}^0 \Delta\phi}{2\pi} e^{\beta_1(x+\delta)+\beta_2 y} \left[K_0(\beta\sqrt{r^2 + \delta^2 - 2\delta r \cos\theta}) - A_0 K_0(\beta r) - 2\sum_{n=1}^{+\infty} A_n K_n(\beta r) \cos(n\theta) \right],$$

$$g = \frac{\epsilon_{11}^0 \Delta\phi - e_{15}^0 b}{2\pi} e^{\beta_1(x+\delta)+\beta_2 y} \left[K_0(\beta\sqrt{r^2 + \delta^2 - 2\delta r \cos\theta}) - A_0 K_0(\beta r) - 2\sum_{n=1}^{+\infty} A_n K_n(\beta r) \cos(n\theta) \right], \quad (r \geqslant R)$$

186   (26)

187 where A_0, A_1, A_2, ... are unknown constants to be determined.
188   By enforcing the traction-free and insulating boundary conditions $\sigma_{zr} = D_r = 0$ (or equivalently $f = g = 0$)
189 at $r = R$, and observing the following Graf's addition theorem (Chew, 1995)
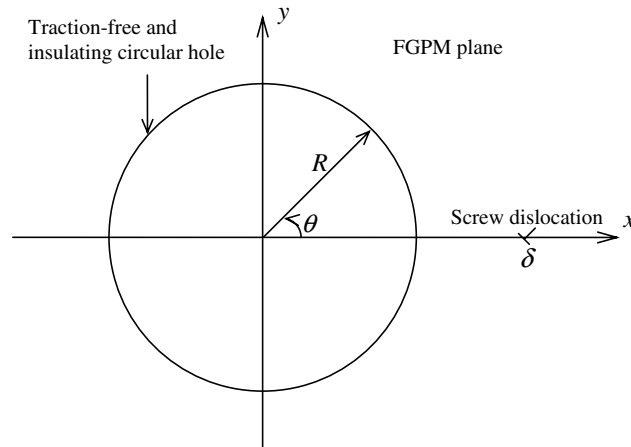
Fig. 2. Interaction between a piezoelectric screw dislocation and a circular insulating hole of radius R in an FGPM plane described by Eq. (4).

$$K_0\left[\beta\sqrt{r^2 + \delta^2 - 2\delta r\cos\theta}\right] = K_0(\beta\delta)I_0(\beta r) + 2\sum_{n=1}^{+\infty} K_n(\beta\delta)I_n(\beta r)\cos(n\theta), \quad \text{for } r < \delta \tag{27}$$

where I_n is the nth order modified Bessel function of the first kind, we can finally arrive at all the unknowns

$$A_0 = \frac{K_0(\beta\delta)I_0(\beta R)}{K_0(\beta R)}, \quad A_n = \frac{K_n(\beta\delta)I_n(\beta R)}{K_n(\beta R)}, \quad n = 1, 2, 3, \ldots, +\infty, \tag{28}$$

In practice the series in Eq. (26) is truncated at a large integer $n = N$ to get sufficiently accurate result.

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