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Microwave Diffraction Measurements in a Parallel-Plate Region*

RONALD V. ROW
Cruft Laboratory, Harvard University, Cambridge, Massachusetts
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Many infinite two-dimensional electromagnetic-scattering problems are strictly equivalent to the configuration obtained from the image principle by placing a finite-scattering obstacle between two parallel, infinite, and perfectly conducting planes and illuminating it with a plane or cylindrical incident wave confined to the parallel-plate region.

An experimental approximation to this arrangement is described for use in the 3-centimeter wavelength region. Finite rectangular parallel metal plates support a transverse incident electromagnetic wave with the electric field oriented perpendicular to the plates. Absorbing wedges at the boundary of the region reduce reflections from this discontinuity. The electromagnetic field scattered from any particular obstacle may be investigated by means of a dipole probe introduced into the region through a moveable section of one of the plates.

The success of this technique is demonstrated by comparing measurements of the total field scattered by infinite conducting wedges of included angle 0, 45, and 90 degrees with the known theoretical expressions, for perpendicular incidence. Agreement with theory is good.

The effect of finite thickness of the diffracting screen (case of 0° wedge) in increasing the amplitude of the scattered wave is noted and compared to theoretical results derived for scattering from a thin conducting half-plane with a cylinder superimposed on the edge.

I. INTRODUCTION

SEVERAL two-dimensional electromagnetic scattering problems involving a plane wave incident on an infinite obstacle with essentially cylindrical symmetry have been investigated theoretically through the use of scalar wave fields, and a complete list of the theoretical contributors to these problems would take too much space here. Using modern microwave techniques many of these problems have been investigated experimentally[1–6] for obstacles as large as several wavelengths in cross-sectional dimension. The experimental arrangements used necessarily involve several approximations to the idealized theoretical model. Kodis[3] has pointed out the advantages of using a single-image plane to isolate the experimenter and measuring equipment from the field being measured. In the same paper, Kodis points out that the non-uniform illumination of a large rectangular screen (approximating a half-plane) occasioned by the use of a point source does not give measurements in good agreement with the well-known theory for this case. This lack of agreement between theory and experiment has led to the development of a parallel-plate region in which both the obstacle and source are, essentially, infinite in extent by virtue of the electromagnetic-image principle.

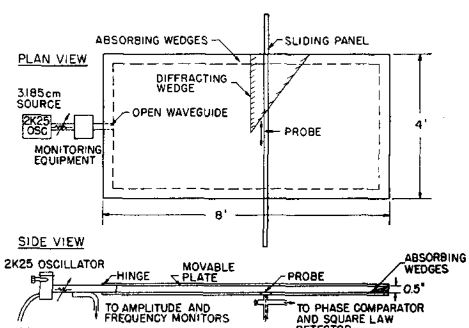

FIG. 1. Schematic layout of parallel-plate region and microwave components.

II. DESCRIPTION OF APPARATUS

The infinite parallel-plate region is approximated experimentally as shown in Fig. 1 by two-plane, parallel 4-ft by 8-ft rectangular duralumin sheets (38 by 76 wavelengths at an operating wavelength of 3.185 cm) spaced $\frac{1}{2}$-in. apart with absorbing wedges inserted along the edges. A line source is approximated by the imaging of the open end of a standard 1-in. by $\frac{1}{2}$-in. X-band wave guide in the parallel plates, and fed from a 1000 cycles/sec square-wave modulated 2K25 reflex-klystron. The spacing of the plates is 0.40 wavelength and is less than half a free-space wavelength, even allowing for spacing variations of 0.020 in., so that the incident wave will be pure transverse electromagnetic in the region occupied by the obstacle and measuring probe. The probe is a dipole antenna less than $\frac{1}{4}$-wavelength long formed by the continuation of the center conductor of a low-impedance coaxial line, and responds to the electric field perpendicular to the plates. It is inserted into the region through a 0.034-in. hole in a sliding panel section of the lower plate and may be moved a distance of 19

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[1] C. W. Horton and R. B. Watson, J. Appl. Phys. 21, 16–21 (1950).
[2] B. N. Harden, Proc. Inst. Elec. Engrs., Pt. III, 99, 229–235 (1952).
[3] R. D. Kodis, J. Appl. Phys. 23, 249–255 (1952).
[4] M. J. Ehrlich, University of California Antenna Laboratory Report (June, 1951).
[5] Esau, Ahrens, and Kebbel, Hochfrequenz. und Electroakustik 53, 113–115 (1939).
[6] W. E. Groves, University of California Antenna Laboratory Report 179, series 7 (1952).

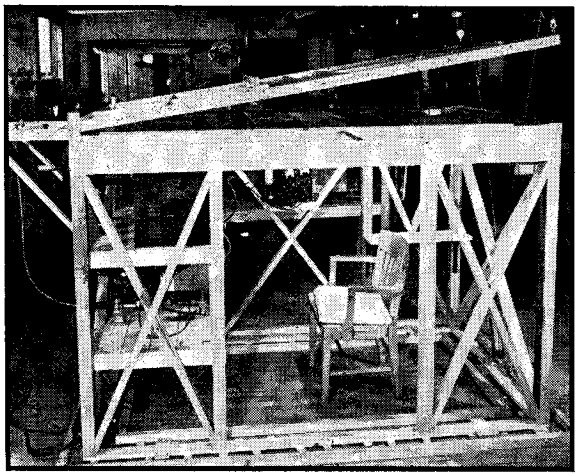

FIG. 2. General view of equipment with parallel plates open.

wavelengths along a straight line; its position being measured within 0.01 cm by means of a scale and vernier. The coaxial line from the probe drives one arm of a wave-guide magic-T phase comparator.[7] For amplitude measurements, the phase-reference signal is disconnected and the detected signal from one side arm of the magic-T amplified. This equipment allows phase and amplitude measurements accurate, at best, to within three degrees and 0.2 db, respectively, over an amplitude range of approximately 40 db. The measuring equipment is located underneath the parallel plate region as shown in Fig. 2.

The absorbing wedges located at the edge of the parallel plate region are made of maple wood coated with colloidal carbon, and reduce the voltage standing-wave ratio in the region traversed by the probe to about 1.05. Measurements of the amplitude and phase distribution from the source, show it to be closely approximated by a line source with its phase center placed approximately in the center of a short side of the region at a point 39.7 wavelengths from the line of probe travel, and this spacing was used in subsequent measurements.

The element of a conducting wedge inserted between the parallel plates is made by forming 0.003-in. silver foil over the edge of a $\frac{1}{2}$-in. thick polyfoam wedge. This resilient structure helps to insure good electrical contact between the wedge and parallel plates, and as an additional precaution, the contacting surfaces of the wedge were coated with a slow-drying silver paint.

III. COMPARISON OF THEORY AND MEASUREMENTS

As an example of its application, the apparatus just described has been used to measure the relative amplitude of the total electric field along a line parallel to and 4 wavelengths behind the illuminated face of conducting wedges of included angles 0, 45, and 90 degrees.

[7] T. Morita and L. S. Sheingold, "A Coaxial Magic-T," Cruft Laboratory Report No. 162, Harvard University (October, 1952).

The theoretical solution to this problem is well known.[8,9] Using the time dependence $e^{-i\omega t}$, the electric field $E_z(x, y)$ satisfies the two-dimensional wave equation,

$$(\nabla_{x,y}^2+k^2)E_z(x,y)=0, \quad k=2\pi/\lambda, \tag{1}$$

subject to the boundary condition that E_z vanish on the surfaces of the wedge, and that the scattered field satisfy the radiation condition at infinity. For a plane wave incident normally on a wedge as shown in Fig. 3, Macdonald[8] gives a solution in terms of the Bessel functions of the first kind,

$$E_z(r,\theta)=\frac{4\pi}{\theta_0}\sum_{n=1}^{\infty}\sin\frac{n\pi^2}{2\theta_0}\times\left(\exp-i\frac{n\pi^2}{2\theta_0}\right)J_{n\pi/\theta_0}(kr)\sin\frac{n\pi}{\theta_0}\theta. \tag{2}$$

Using Sommerfeld's contour integral representation for the Bessel function $J_{n\pi/\theta_0}(kr)$ and interchanging the order of summation and integration, Macdonald's integral formula is obtained. For large kr, the method of steepest descents applied to this integral yields the following approximate formula for the field,

$$E_z(r,\theta)=\frac{\exp(-i\frac{1}{4}\pi+ikr)}{2\theta_0}\left(\frac{2\pi}{kr}\right)^{\frac{1}{2}}\sin\frac{\pi^2}{\theta_0}\times\left(\frac{1}{\cos\pi^2/\theta_0-\cos\pi/\theta_0(\theta-\pi/2)}-\frac{1}{\cos\pi^2/\theta_0-\cos\pi/\theta_0(\theta+\pi/2)}\right). \tag{3}$$

This formula is not valid for angles near $\theta=3\pi/2$, i.e., along the light-shadow boundary, but away from this

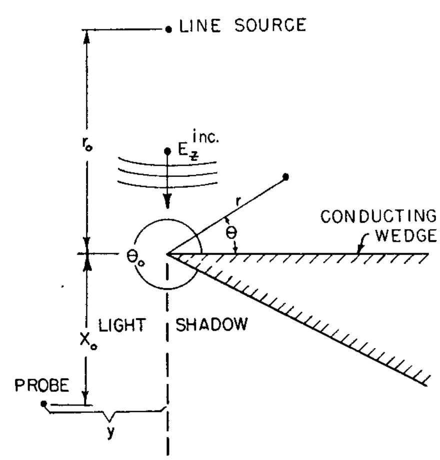

FIG. 3. Coordinate system for scattering from a wedge.

[8] H. M. Macdonald, *Electric Waves* (Cambridge University Press, Cambridge, 1902), p. 186.

[9] A. Sommerfeld, Math. Ann. **45**, 263 (1894).

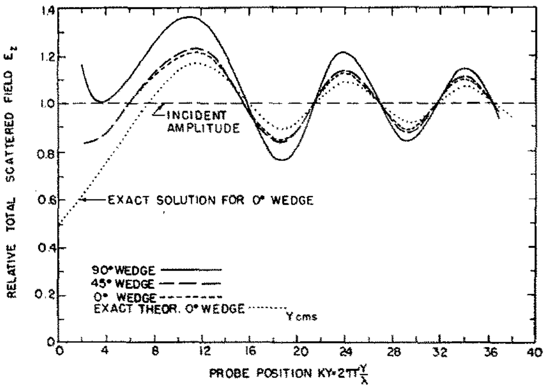

FIG. 4. Theoretical scattered field from wedges of various angles, computed from the saddle-point formula. (Phase corrected for a line source.)

boundary it shows the correct trend in the field as the angle θ_0 of the wedge is varied (see Fig. 4).

With a wedge of angle $\theta_0=2\pi$, the problem reduces to the well-known half-plane problem for which we have Sommerfeld's[9] solution,

$$E_z(r,\theta)=\exp(-ikr\sin\theta)+\frac{\exp(-i\frac{1}{4}\pi)}{\sqrt{\pi}}\exp(-ikr\sin\theta)\times\int_0^{(2kr)^{\frac{1}{2}}\cos\frac{1}{2}(\theta-\pi/2)}(\exp i\zeta^2)d\zeta, \quad (4)$$

valid in the region $\pi/2<\theta\leq 3\pi/2$.

In this last formula, the first term, $\exp-ikr\sin\theta=\exp-ikx$, represents the incident plane wave, and the second term represents the wave scattered by the half-plane.

Carslaw[10] has solved the same boundary value problem for a line-source of primary excitation. His results show for a large source to screen separation and for points not too far from the edge of the screen, that the scattered field is the same as the scattered field for plane-wave incidence multiplied by the factor $1/\sqrt{r_0}$, where r_0 is the separation between the line source and the edge. In making measurements the field at the point $x=x_0=4\lambda$, $y=0$ was used as a reference, hence the field at the edge is greater than this reference field by the factor $(r_0+x_0/r_0)^{\frac{1}{2}}$. Thus, in plotting the theoretical results for comparison with the relative measured field in Figs. 5 and 6 the formula

$$|E_z(r,\theta)|=\left|e^{i\phi(y)}+\left(\frac{r_0+x_0}{r_0}\right)^{\frac{1}{2}}\frac{\exp-i\frac{1}{4}\pi}{\sqrt{\pi}}\times\int_0^{(2kr)^{\frac{1}{2}}\cos\frac{1}{2}(\theta-\frac{1}{2}\pi)}(\exp i\zeta^2)d\zeta\right|, \quad (5)$$

was used. Where $\phi(y)$ is the phase corresponding to a line source located 35.7 wavelengths from the diffracting edge along a line perpendicular to the screen and passing through the edge. The integral in Eq. (5) is readily computed using tables of the Fresnel Integrals.

For the half-plane case there is excellent agreement

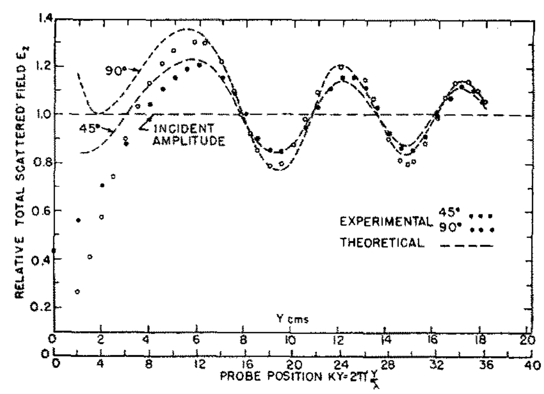

FIG. 6. Approximate theoretical and measured diffraction patterns from 45 and 90 wedges (line source).

between theory and measurements, certainly within the limits of reproducibility of the data. For the 45 and 90 degree wedges the theory is only approximate, and the experimental results confirm the trend predicted by theory.

IV. EFFECT OF EDGE THICKNESS

The exact theory for diffraction by a half-plane considers only the case where the plane is of zero thickness. However, the screen used in any experiment has some thickness, and it is important to know just what effect this thickness will have on the measurements. From simple physical considerations it is to be expected that regardless of its precise shape, a small increase in the thickness of the edge of a very thin screen will result in a corresponding linear increase in the amplitude of the scattered wave. Figure 7 shows results measured with rectangular edges of thickness 0.0024 wavelength, 0.125 wavelength, and 0.250 wavelength. In addition the exact theoretical result for the ideal half-plane is shown

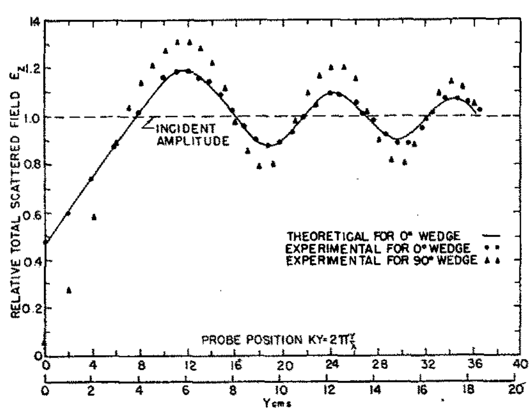

FIG. 5. Measured diffraction patterns from 0° and 90° wedges (line source).

[10] H. S. Carslaw, Proc. London Math. Soc. (1) 30, 121 (1899).

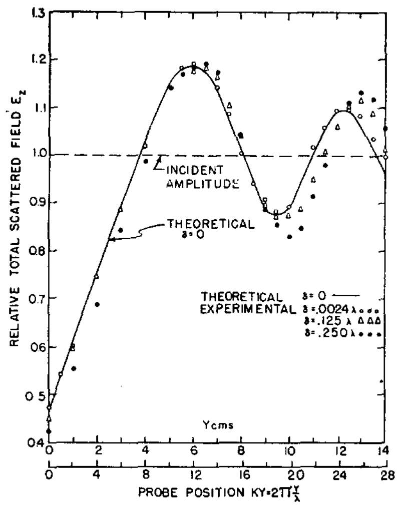

FIG. 7. Measured diffraction from a rectangular edge for different thicknesses (line source).

for comparison. These indeed show an increase in the scattered wave, and a slight shift in the location of maxima and minima as the thickness increases above 0.125 wavelength. The successful theoretical analysis of diffraction by a rectangular edge remains as an unsolved problem.† However, Karp[11] has obtained the following solution to the problem of diffraction by a half-plane with a cylinder superimposed on the edge as shown in Fig. 8.

For a plane wave incident normally on a half plane the series solution (2) reduces to

$$E_z(r,\theta)=2\sum_{n=1}^{\infty}\sin\frac{n\pi}{4}e^{-in\frac{1}{4}\pi}J_{n/2}(kr)\sin\frac{n\theta}{2}.$$

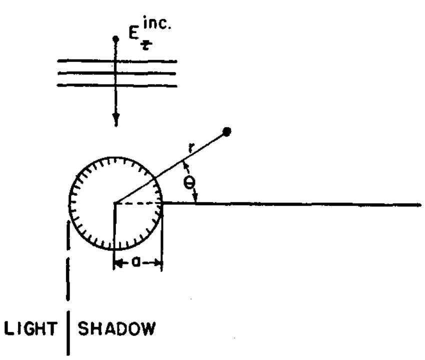

FIG. 8. Coordinate system for scattering from a half plane with a cylindrical edge (line source).

† In a recent paper D. S. Jones [Proc. Roy. Soc. (London) **A217**, 153 (1953)] gives the solution to this problem. However, no results were given in a form allowing comparison with the measurements made here.

[11] S. N. Karp, Progress Reports Nos. 8, 9 and 10, Washington Square College of Arts and Sciences, New York University.

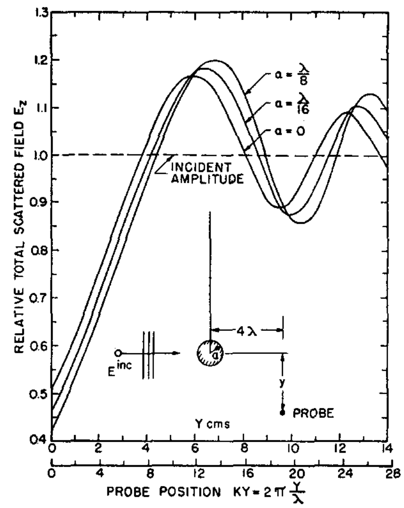

FIG. 9. Theoretical diffraction pattern from a half plane with a cylindrical edge (phase corrected for a line source).

If from this result the series,

$$E_c(r,\theta)=2\sum_{n=1}^{\infty}\sin\frac{n\pi}{4}e^{-in\frac{1}{4}\pi}\times\frac{J_{n/2}(ka)}{H_{n/2}^{(1)}(ka)}H_{n/2}^{(1)}(kr)\sin\frac{n\theta}{2},\quad(6)$$

is subtracted, the resultant field

$$E_z^{\mathrm{tot}}(r,\theta)=E_z(r,\theta)-E_c(r,\theta),\quad(7)$$

satisfies the wave equation and vanishes on the conducting boundary surfaces.

Figure 9 shows a plot of Eq. (7) for cylinder radius $a=0$, $\lambda/16$ and $\lambda/8$, as the point of observation moves along a straight line parallel to and 4 wavelengths

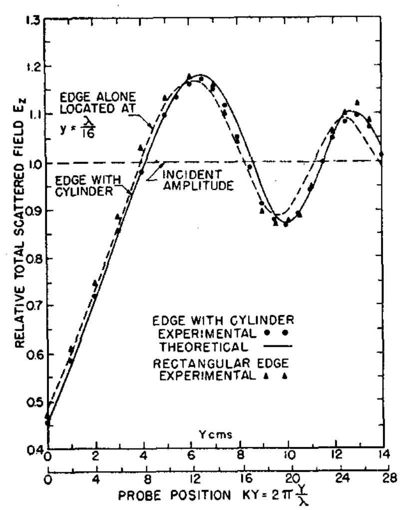

FIG. 10. Theoretical and measured diffraction from a half plane with a circular-cylindrical edge of radius=/16 and a rectangular edge of thickness=/8 (line source).

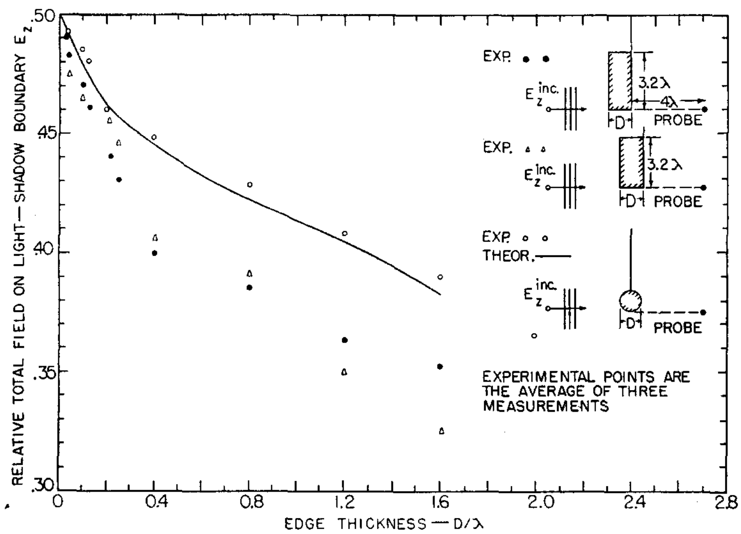

FIG. 11. Measured and theoretical field on the light-shadow boundary for a half plane with a cylindrical edge and measured field for a half plane with a rectangular edge (phase corrected for a line source).

behind the illuminated side of the screen. Figure 10 shows a comparison between theory and measurements for diffraction by a half plane with a cylindrical a swell as a rectangular edge of $\frac{1}{8}$-wavelength diameter.

For $ka<0.1$ the first term of the series (6) provides an accurate correction (within about 1 percent) to the field from the half-plane alone. For the field on the light-shadow boundary four wavelengths behind the screen this correction gives,

$$|E_z^{\mathrm{tot}}| \simeq \tfrac{1}{2} - \frac{ka}{2\pi} + \left(\frac{ka}{2\pi}\right)^2 + \cdots. \tag{8}$$

This formula should also give a good indication of the behavior of the field on the light-shadow boundary in the case of diffraction by a thin screen with a rectangular edge of thickness $2a$. Figure 11 presents theoretical and experimental results for the relative total field on the light-shadow boundary for the circular edge, and in addition experimental results for the rectangular edge. As expected, for ka small ($ka<0.03$) there is agreement, within the reproducibility of the measurements, between the measured field on the light-shadow boundary with both the rectangular and circular edge, and the theory for the circular edge. The agreement between theory and experiment for the circular edge is good over the entire range of cylinder diameters measured.

V. CONCLUSIONS

The use of a parallel plate region provides an excellent approximation to an ideal two-dimensional space for the experimental study of a number of electromagnetic scattering problems. Measurements are, of course, limited to cases where (1) the incident electric field is oriented perpendicular to the parallel plates and (2) the obstacle is periodic in at least one direction. The problem of obtaining a closer approximation to plane-wave excitation than that afforded by the present line source still remains unsolved. However, even within this limitation, the apparatus remains a useful companion tool with theory in further investigations of scattering problems in the region between the quasistatic and optical limits. Investigations of the scattering by closely coupled circular cylinders and a pair of parallel slits in two parallel screens are now active.

ACKNOWLEDGMENT

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