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Calculation of sound radiant efficiency and sound radiant modes of arbitrary shape structures by BEM and general eigenvalue decomposition

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Abstract

In this paper, a numerical method is presented to calculate sound radiant efficiency and radiant modes of arbitrary shape structures. Some methods have been proposed to compute sound radiant efficiencies and sound radiant modes of plates and beams. However, there is not a valid method to calculate for arbitrary shape structures except for measurement at the present time. The method proposed can predicate the sound radiant efficiencies and the sound radiant modes for arbitrary shape structures by boundary element method (BEM) and general eigenvalue decomposition. The validity of this method is demonstrated by two numerical examples of pulsating sphere and radiation cube.

Keywords: Sound radiation; Sound radiant mode; Boundary element method; Sound radiant efficiency

1. Introduction

The prediction and control of acoustic radiation become a more and more important issue in new product design process. The sound radiant efficiency and sound power are important parameters to describe the radiant characteristic of structures. However, it is well known that the total sound power can not be directly calculated by the modal efficiencies when radiant efficiencies are described by structural modes shape function. The main reason is that the radiation of structures modes is strongly coupled each other, which is also the main difficulty of controlling and computing sound radiation. Many scholars have studied the sound radiation behavior and tried to find the connection between the sound radiation and vibration, and to find an approach to decouple the sound radiation.

Photiadis [1], Cunefare et al. [2,3], Currey and Cunefare [4] and Elliott [5] presented the concept of the acoustical radiant modes during early 1990s. The advantage of this approach is the elimination of the complexity of the structure mode's coupling terms. The sound power of structures can be expanded as the sum of sound radiant modes. For plate and beam, Photiadis [1], Cunefare et al. [2,3], Currey and Cunefare [4] have studied their radiant modes and radiant efficiencies using the concept of the acoustical radiant modes. Snyder and Tanaka [6] clarified the coupling reason of sound radiation by structures modes calculation. As an example, he analyzed the radiation characteristic of radiant modes of thin plates. Oppenheimer and Dubowsky [7] tested the radiant efficiency of plates by experiment. Elliott [5] and Koorosh et al. [8,9] applied this idea to study the active control of the sound radiation. The results showed that this method could obtain notable effect.

However, for complex structures, such as arbitrary enclosing shape structures, there is not a valid method to compute the sound radiant efficiency and sound radiant

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modes except for measurement. Hashimoto [10] have presented a method to measure sound radiant efficiency.

In this paper, a numerical approach is proposed to calculate the sound radiant efficiency and sound radiant modes of arbitrary shape structures by boundary element method and general eigenvalue decomposition.

Boundary element method (BEM) is a validity method to compute structure-born acoustical radiation of complex structures [11–15]. In this paper, the surface pressure of structures is computed via BEM, and then the sound power can be expressed as a Hermitian quadratic form, and the equivalent sound power can be expressed as a quadratic form. As the impedance matrix of sound power of a structures is positive definite, and the coupling matrix of mean square of the equivalent sound power is real symmetrical and positive definite, the sound radiation can be decoupled via general eigenvalue decomposition, the radiant modes of structures are the eigenvectors of eigenvalues. The sound radiant efficiencies and sound radiant power can be expanded as the sum of the sound radiant modes if radiant modes are taken as a basis.

2. Basic boundary element formulation

Consider the acoustic pressure field in the exterior unbound domain. The governing differential equation of the exterior acoustic domain in steady-state linear acoustics is the classical Helmholtz equation [11] as follows

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where p is the sound pressure, $k = \omega/c$ is the wavenumber, ω and c are the angular frequency and speed of sound, respectively.

The Neumann boundary condition and the Sommerfeld radiation condition at infinity can be expressed as

$$\frac{\partial p}{\partial n} = -\mathrm{i}\rho\omega v_n \tag{2}$$

$$\lim_{r\to\infty}\left[r\left(\frac{\partial p}{\partial r} - \mathrm{i}kp\right)\right] = 0 \tag{3}$$

where $\mathrm{i} = \sqrt{-1}$, and ρ is the density of the fluid.

For arbitrary enclosed structures, including those with corners and edges, the form of Helmholtz integral equation [11,13] is given by

$$C(P)p(P) = \int_S \left(p\frac{\partial G}{\partial n} + \mathrm{i}\rho\omega v_n G\right)\mathrm{d}S \tag{4}$$

where n is the unit normal vector pointing into the acoustic domain, and G is the fundamental solution of the inhomogeneous Helmholtz equation, namely the Green's function of free space, and $\partial G/\partial n$ is derivative of G with respect to n. They can be written as follows:

$$\begin{cases} G(r) = \frac{e^{-\mathrm{i}kr}}{r} \\ \frac{\partial G}{\partial n} = -\left(\mathrm{i}k + \frac{1}{r}\right)G\frac{\partial r}{\partial n} \end{cases} \tag{5}$$

where $r = |\bar{x}_P - \bar{x}_Q|$, and x_Q is any point in space. For any point P in space, the value of coefficient $C(P)$ can be expressed as [13]

$$C(P) = \begin{cases} 1 & P \in V \\ 4\pi + \int_S \frac{\partial}{\partial n}\left(\frac{1}{r}\right)\mathrm{d}S & P \in S \\ 0 & P \in (V \cup S) \end{cases} \tag{6}$$

The evaluation of Eq. (4) was performed using isoparametric element and numerical quadrature. For isoparametric element, interpolation of the pressure and velocity at the nodes determines the pressure and velocity distributions over entire element [16]

$$\begin{cases} p = \sum_{l=1}^{4} N_l p_l \\ v = \sum_{l=1}^{4} N_l v_l \end{cases} \tag{7}$$

where $p_l = (p_1, p_2, p_3, p_4)^{\mathrm{T}}$, $v_l = (v_1, v_2, v_3, v_4)^{\mathrm{T}}$,$p_l$ and v_l represent the pressure and velocity of nodes of element, respectively, and N_l is an interpolation shape function

$$N_l = \frac{1}{4}(1 + \xi_l\xi)(1 + \eta_l\eta) \quad l = 1, 2, 3, 4 \tag{8.1}$$

The exactly formula of the shape function is

$$N_1 = \frac{1}{4}(1 + \xi)(1 + \eta) \tag{8.2}$$

$$N_2 = \frac{1}{4}(1 - \xi)(1 + \eta) \tag{8.3}$$

$$N_3 = \frac{1}{4}(1 - \xi)(1 - \eta) \tag{8.4}$$

$$N_4 = \frac{1}{4}(1 + \xi)(1 - \eta) \tag{8.5}$$

where ξ and η is local coordinate.

The numerical evaluation of Eq. (4) for the nodes of each element ultimately yields a system of algebraic equation [16]

$$H_H P = G_H V \tag{9}$$

so

$$P = (H_H^{-1} G_H)V = ZV \tag{10}$$

where $P = (p_1, p_2, \ldots, p_n)^{\mathrm{T}}$, $V = (v_1, v_2, \ldots, v_n)^{\mathrm{T}}$, $Z = H_H^{-1} G_H$.

It is well known that the classical boundary element method for exterior acoustical problem fails to provide a unique solution at certain frequency. Several modified integral formulations have been developed to overcome the problem. By far, the CHIEF method [11,15] is the most popular method, and it is adopted in this study.

For exterior sound radiation problem, if one chooses some CHIEF points out of domain V and S, these points, according to Eq. (6), satisfy the Helmholtz equation [11]

$$\int_S \left(p\frac{\partial G}{\partial n} + \mathrm{i}\rho\omega v_n G\right)\mathrm{d}S = 0 \tag{11}$$

Introduce Eq. (7) into Eq. (11), one may get

$$H_cP + G_cV = 0 \quad (12)$$

Eq. (12) and Eq. (9) form the following simultaneous equations

$$\begin{bmatrix} H_H \\ H_C \end{bmatrix} P - \begin{bmatrix} G_H \\ -G_C \end{bmatrix} V = 0 \Rightarrow D_{(N+M)\times N} P_{N\times 1} = B_{(N+M)\times N} V_{N\times 1} \quad (13)$$

Eq. (13) is an over determinate equation. Since $D^{-1}_{N\times(M+N)}$ is the Moore–Penrose pseudoinverse of matrix $D_{(M+N)\times N}$, the following equation can be got

$$P_{N\times 1} = D^{-1}_{N\times(N+M)} B_{(N+M)\times N} V_{N\times 1} = A_{N\times N} V_{N\times 1} \quad (14)$$

where $A_{N\times N} = D^{-1}_{N\times(N+M)} B_{(N+M)\times N}$, $P = (p_1, p_2, \ldots, p_n)^{\mathrm{T}}$, $V = (v_1, v_2, \ldots, v_n)^{\mathrm{T}}$.

3. Sound power of arbitrary shape structures

For an arbitrary shape structures, its whole sound power can be expressed as

$$W = \int_S I_s \, \mathrm{d}S \quad (15)$$

Especially, for simple harmonic vibration

$$I_s = \frac{1}{2}\mathrm{Re}\left[P^{\mathrm{T}}(Q)V_n^*(Q)\right] \quad (16)$$

where Q are the element nodes of enclosing structures, I_S and P are the sound intensity and pressure of nodes, respectively, $V_n(Q)$ is the normal velocity of node Q. * (asterisk) indicates complex conjugation of the quantity. $P = (p_1, p_2, \ldots, p_N)^{\mathrm{T}}$, $V = (v_1, v_2, \ldots, v_N)^{\mathrm{T}}$.

If the structure is divided into N_e elements, the sound power can be rewritten as

$$W = \frac{1}{2}\mathrm{Re}\sum_{i=1}^{N_e} \int_{S_i} (pV^*)\mathrm{d}S_i \quad (17)$$

Introduce a notation

$$W_i = \int_{S_i} (pV^*)\mathrm{d}S_i \quad (18)$$

According to the Eq. (7), the sound pressure and the velocity on each element is approximated by a linear combination of shape function via four nodes, leads Eq. (7) into Eq. (18), the following equation can be get

$$W_i = \int_{S_i} \sum_{l=1}^{4} (N_l p_l)^{\mathrm{T}} \sum_{l=1}^{4} (N_l v_l^*)\mathrm{d}S_i = \int_{S_i} P_{\mathrm{el}}^{\mathrm{T}} N^{\mathrm{T}} N V_{\mathrm{el}}^* \, \mathrm{d}S_i \quad (19)$$

where P_{el} and V_{el} denote the sound pressure and the normal velocity of the four nodes of element S_i, and $P_{\mathrm{el}} = [p_{i1} \quad p_{i2} \quad p_{i3} \quad p_{i4}]^{\mathrm{T}}$, $V_{\mathrm{el}} = [v_{i1} \quad v_{i2} \quad v_{i3} \quad v_{i4}]^{\mathrm{T}}$; N is shape function, and $N = [N_1 \quad N_2 \quad N_3 \quad N_4]$.

According to Eq. (14), the sound pressure of node j can be written as

$$P_j = P_j^{\mathrm{T}} = (A_j V)^{\mathrm{T}} = V^{\mathrm{T}} A_j^{\mathrm{T}} \quad (20)$$

where A_j is the jth column of matrix A.

Substitute Eq. (20) into Eq. (19), one may get

$$W_i = V^{\mathrm{T}} \int_{S_i} (A_{\mathrm{el}}^{\mathrm{T}} N^{\mathrm{T}} N)\mathrm{d}S_i V_{\mathrm{el}}^* \quad (21)$$

where $A_{\mathrm{el}} = [A_{i1} \quad A_{i2} \quad A_{i3} \quad A_{i4}]$, and A_{i1} denotes the jth column of matrix A corresponding the node number of sound pressure p_j in global coordinate.

Introduce a notation

$$C_i = \int_{S_i} (A_{\mathrm{el}}^{\mathrm{T}} N^{\mathrm{T}} N)\mathrm{d}S_i \quad (22)$$

Eq. (21) can be rewritten as

$$W_i = V^{\mathrm{T}} C_i V_{\mathrm{el}}^* \quad (23)$$

Introduce Eq. (23) into Eq. (17), one may get

$$W = \frac{1}{2}\mathrm{Re}\sum_{i=1}^{N_e} W_i = \frac{1}{2}\mathrm{Re}\sum_{i=1}^{N_e} (V^{\mathrm{T}} C_i V_{\mathrm{el}}^*) = \frac{1}{2}\mathrm{Re}(V^{\mathrm{T}} C V^*) \quad (24)$$

According to the fact that $V^{\mathrm{T}}CV^*$ is a factor, the sound power can be rewritten as

$$W = \frac{1}{4}[(V^{\mathrm{T}}CV^*) + (V^{\mathrm{T}}CV^*)^*] \quad (25)$$

So

$$W = \frac{1}{4}\{(V^{\mathrm{T}}CV^*)^{\mathrm{T}} + (V^{\mathrm{T}}CV^*)^*\} = V^{\mathrm{H}}\left(\frac{C^{\mathrm{T}} + C^*}{4}\right)V = V^{\mathrm{H}}MV \quad (26)$$

where $M = \frac{C^* + C^{\mathrm{T}}}{4}$, superscript H denotes conjugation transpose.

It is obvious that

$$M^{\mathrm{H}} = \left(\frac{C^* + C^{\mathrm{T}}}{4}\right)^{\mathrm{H}} = \frac{C^{\mathrm{T}} + C^*}{4} = M \quad (27)$$

where M is an impedance matrix. From Eq. (27), the matrix M is a Hermitian matrix. For arbitrary velocity vector, the sound power is positive, and matrix M is positive definite and Hermitian matrix M is nice because the follow reasons

1. It has n positive eigenvalues.
2. The eigenvectors of it form a basis of n dimensional space.

4. General eigenvalue decomposition and sound radiation decoupling

According to the definition of sound radiant efficiency, it can be described as follows [10]

$$\sigma_{\text{rad}} = \frac{W_{\text{rad}}}{\rho_0 c \langle v^2 \rangle S} \tag{28}$$

where W_{rad} is total sound power of structures, σ_{rad} is sound radiant efficiency, $\rho_0 c$ is characteristic impedance of media, $\langle v^2 \rangle$ is the spatial mean-square velocity of the surface of enclosed structures, S is the equivalent area of structures. The definition of spatial mean-square velocity is

$$\langle v^2 \rangle = \frac{1}{2S} \int_S |v|^2 \mathrm{d}S \tag{29}$$

Introduce Eq. (7) into Eq. (29), one may get

$$\langle v^2 \rangle = \frac{1}{2S} \int_S |v|^2 \mathrm{d}S = \frac{1}{2S} \int_S v^{\mathrm{H}} v \, \mathrm{d}S = \frac{1}{2S} \sum_{i=1}^{N_e} v_i^{\mathrm{H}} \int_{S_i} N^{\mathrm{T}} N \, \mathrm{d}S_i v_i \tag{30}$$

Therefore

$$\langle v^2 \rangle = \frac{1}{2S} \sum_{i=1}^{N_e} v_i^{\mathrm{H}} M_i v_i = \frac{1}{2S} V^{\mathrm{H}} M_0 V \tag{31}$$

where N_e is the nodes number of each element, and N is shape functions, $M_i = \int_{S_i} N^{\mathrm{T}} N \, \mathrm{d}S_i$.

Introduce a notation

$$\rho_0 c \langle v^2 \rangle S = \rho_0 c \frac{1}{2S} V^{\mathrm{H}} M_0 V S = V^{\mathrm{H}} \frac{\rho_0 c}{2} M_0 V = V^{\mathrm{H}} M_{\text{st}} V \tag{32}$$

where $M_{\text{st}} = \frac{\rho_0 c}{2} M_0$.

According to Eqs. (26), (28) and (32), the sound radiant efficiency can be expressed as

$$\sigma_{\text{rad}} = \frac{V^{\mathrm{H}} M V}{V^{\mathrm{H}} M_{\text{st}} V} \tag{33}$$

From the definition of spatial mean-square velocity, it can be drawn that the matrix M_{st} is real symmetric and positive definite. Now the sound radiation can be decoupled via general eigenvalue decomposition. According to the characteristic of matrix M_{st}, one can find a nonsingular matrix L which satisfies

$$L^{\mathrm{T}} M_{\text{st}} L = I \tag{34}$$

For M matrix is positive definition, it can be concluded that $L^{\mathrm{T}} M L$ is also positive definition matrix. Therefore, one can find a U matrix which satisfies

$$U^{\mathrm{H}} L^{\mathrm{H}} M L U = \Lambda \tag{35}$$

At the same time

$$U^{\mathrm{H}} L^{\mathrm{H}} M_{\text{st}} L U = I \tag{36}$$

Introduce a notation

$$q = LU \tag{37}$$

The row vectors of matrix q are the sound radiant modes, which can make matrix M and M_{st} diagonal together

$$q^{\mathrm{H}} M q = \Lambda = \mathrm{diag}(\lambda_1, \lambda_1, \ldots, \lambda_N) \tag{38}$$

$$q^{\mathrm{H}} M_{\text{st}} q = I \tag{39}$$

For any velocity distribute V, it can be expressed as

$$V = qa \tag{40}$$

where a is the coordinate of velocity distribution V in vector space composed of sound radiant modes.

Introduce Eq. (38) and Eq. (39) into Eq. (33), σ_{rad} can be expressed below

$$\sigma_{\text{rad}} = \frac{V^{\mathrm{H}} M V}{V^{\mathrm{H}} M_{\text{st}} V} = \frac{a^{\mathrm{T}} q^{\mathrm{H}} M q a}{a^{\mathrm{T}} q^{\mathrm{H}} M_{\text{st}} q a} = \frac{\sum_{i=1}^{N} \lambda_i a^{\mathrm{T}} a}{a^{\mathrm{T}} a} \tag{41}$$

5. Numerical examples

In order to test the accuracy and efficiency of the presented method in the paper, two cases of acoustic radiation problems, pulsating sphere and radiating cube, are tested. The sound power and radiant efficiency are compared to the theoretical results. The sound radiant modes and the radiation efficiencies of the two cases are also showed.

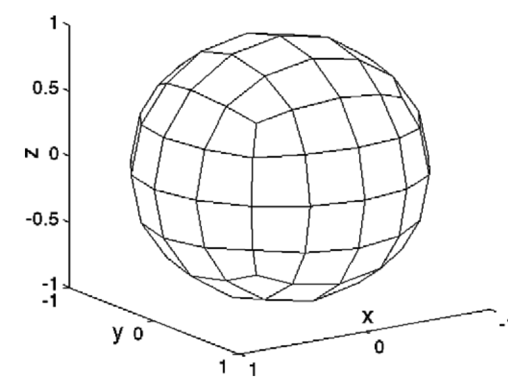

Fig. 1. Model of a pulsating sphere of 96 elements, radius $a = 1$ m.

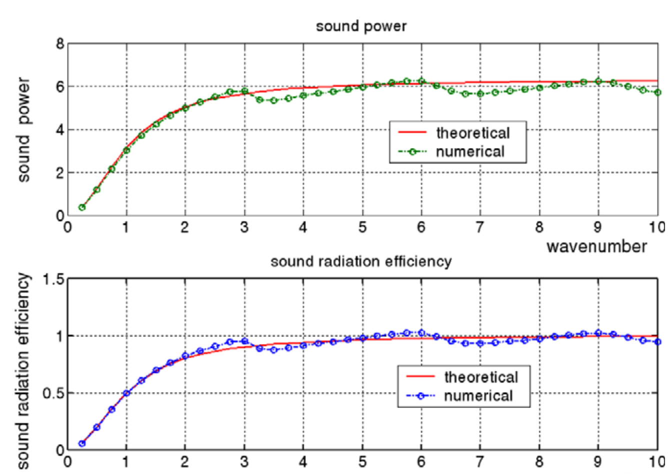

Fig. 2. The results of numerical and theoretical acoustical power and acoustical efficiency.

5.1. A pulsating sphere

The analytical solution of sound pressure $p(r)$, and the radiant efficiency σ, of a pulsating sphere of radius a with an uniform radial velocity, v_{n}, are given by [11]:

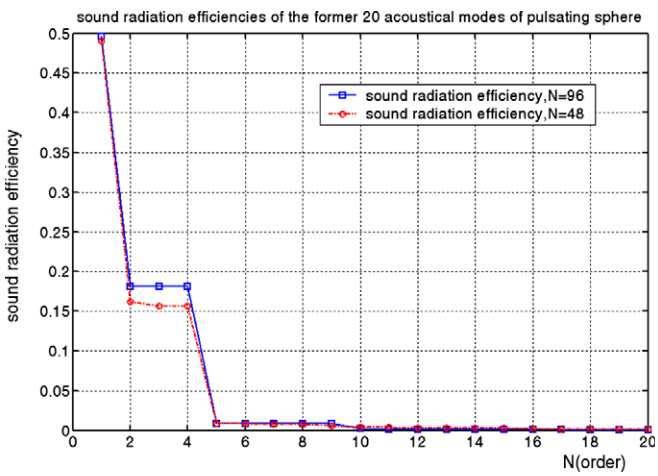

Fig. 3. The former 20 acoustical efficiencies of a pulsating sphere with different mesh when $k = 1$.

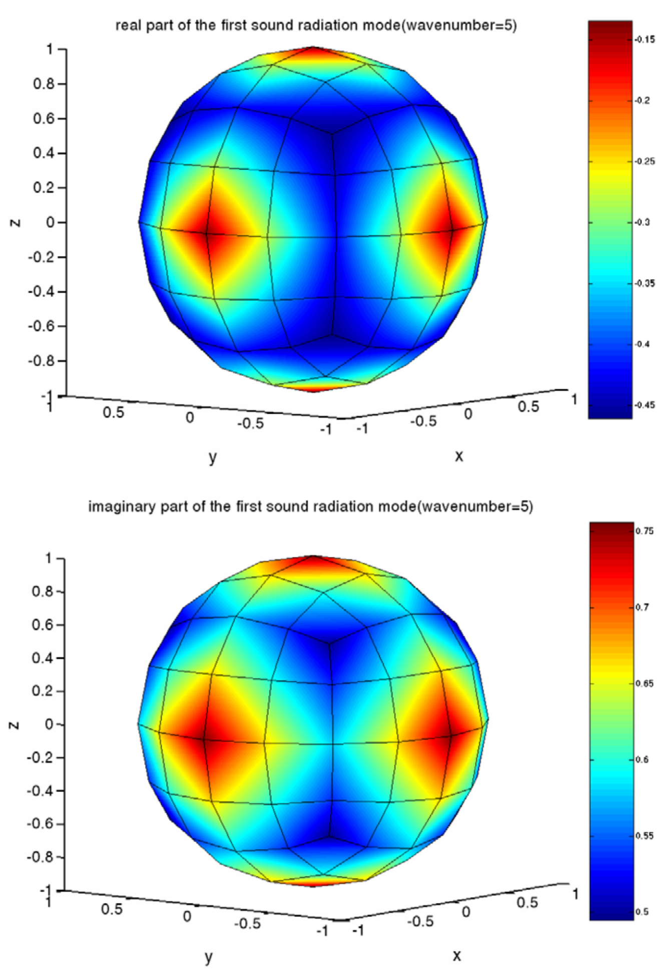

Fig. 4. The first acoustical mode of pulsating sphere.

$$p(r) = \rho c_0 v_n \frac{a}{r} \frac{\mathrm{i}ka}{1 + \mathrm{i}ka} \mathrm{e}^{-\mathrm{i}k(r-a)} \tag{42}$$

$$\sigma = (ka)^2/[1 + (ka)^2] \tag{43}$$

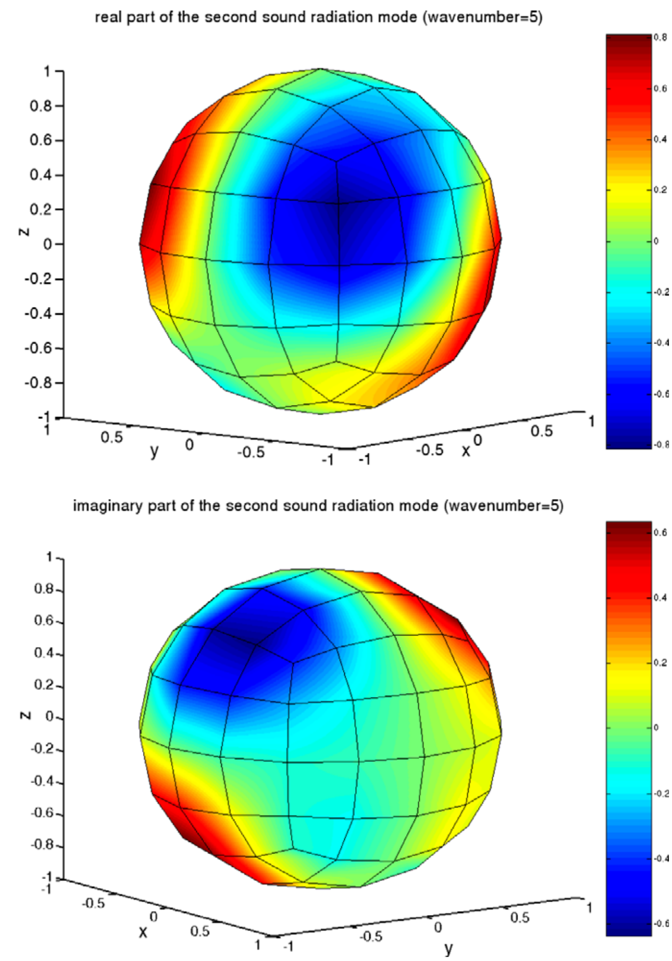

Fig. 5. The second acoustical mode of pulsating sphere.

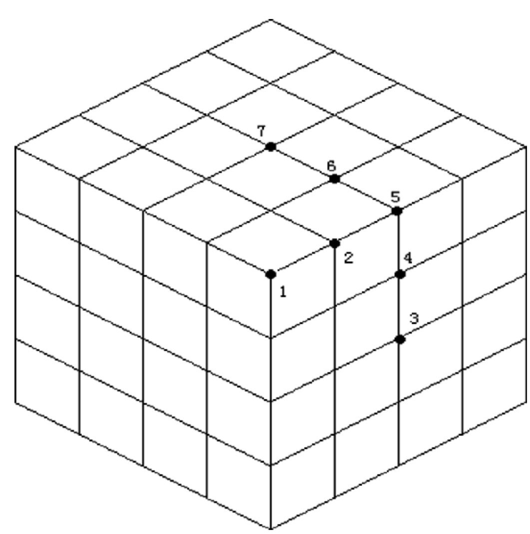

Fig. 6. Mesh model of a radiating cube.

Table 1
Normalized pressure magnitude on the surface of radiating cube for $ka = 1$ with different element size $l_e = 0.5$ m and $l_e = 0.25$ m

	Node 1	Node 2	Nodes 3 and 7	Nodes 4 and 6	Node 5	W	σ
Analytical solution	0.40825	0.47140	0.70711	0.63646	0.5	3.1416	–
$l_e = 0.5$ m	0.39866	0.46200	0.69229	0.62308	0.49011	2.9697	0.7114
$l_e = 0.25$ m	0.40581	0.46900	0.70246	0.62980	0.49747	3.0979	0.7026

The surface of the sphere is modeled using 96 curvilinear quadrilateral isoparametric elements, as depicted in Fig. 1. A few parameters are introduced below for convenience. Radius of the sphere $a = 1$ m, air density $\rho = 1$ kg/m^3, and sound velocity $c_0 = 1$ m/s.

Assume that the pulsating sphere has vibrational amplitude of 1. According to the method presented in the paper, the comparison between numerical sound power and sound radiant efficiency with corresponding theoretical solutions is shown in Fig. 2. The computational results is very closed to theoretical result, therefore, it shows that the method presented in this paper can be utilized to solve both the sound radiation power and sound radiant efficiency by numerical method for the structure with curve surface.

Fig. 3 depicts the former 20 sound radiant efficiencies of radiant modes with $k = 1$. The results show that the first radiant mode can radiate sound power effectively. Because the sphere has three symmetrical directions, the radiant efficiencies from second to fourth radiant modes are equivalent. It can be drawn from Fig. 3 that different velocity distribution maybe has the same sound radiant efficiency.

Figs. 4 and 5 show the former two radiant modes with $k = 5$, respectively. The results show that the sound power mainly comes from the real part, and the different color indicate the different velocity amplitude distribute of the sphere surface. Those results illiterates the sound power mainly comes from the former sound radiant modes.

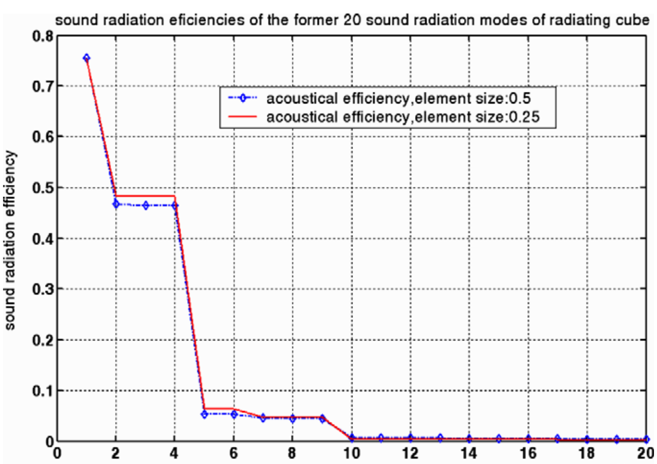

Fig. 7. The former 20 acoustical efficiencies of radiation modes with different element size where $k = 1$.

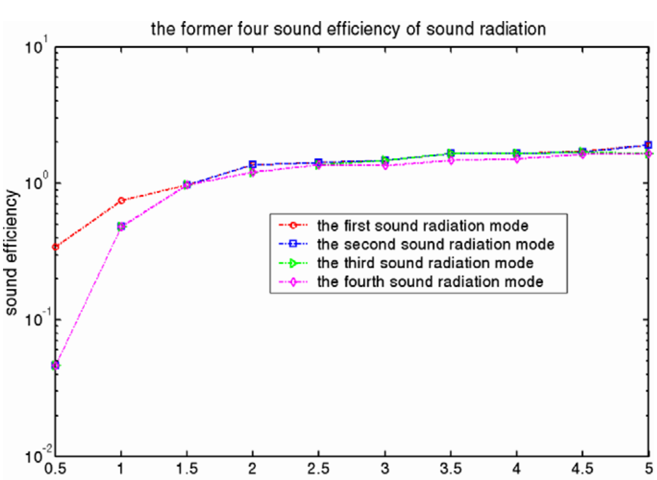

Fig. 8. Sound radiation efficiencies of the former 4 sound radiation modes with different wavenumber.

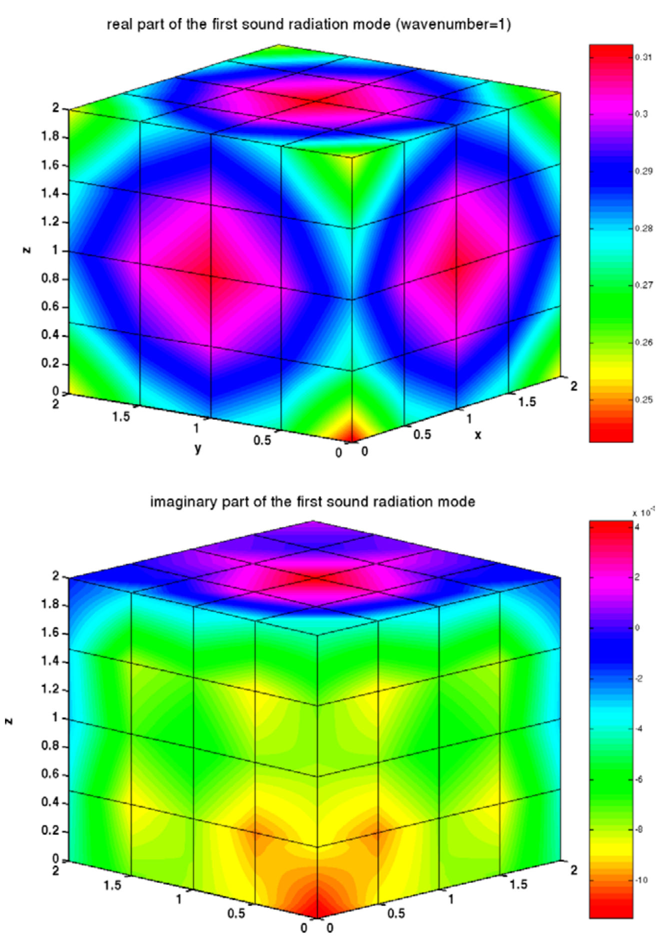

Fig. 9. The first acoustical mode of radiating cube for $k = 1$.

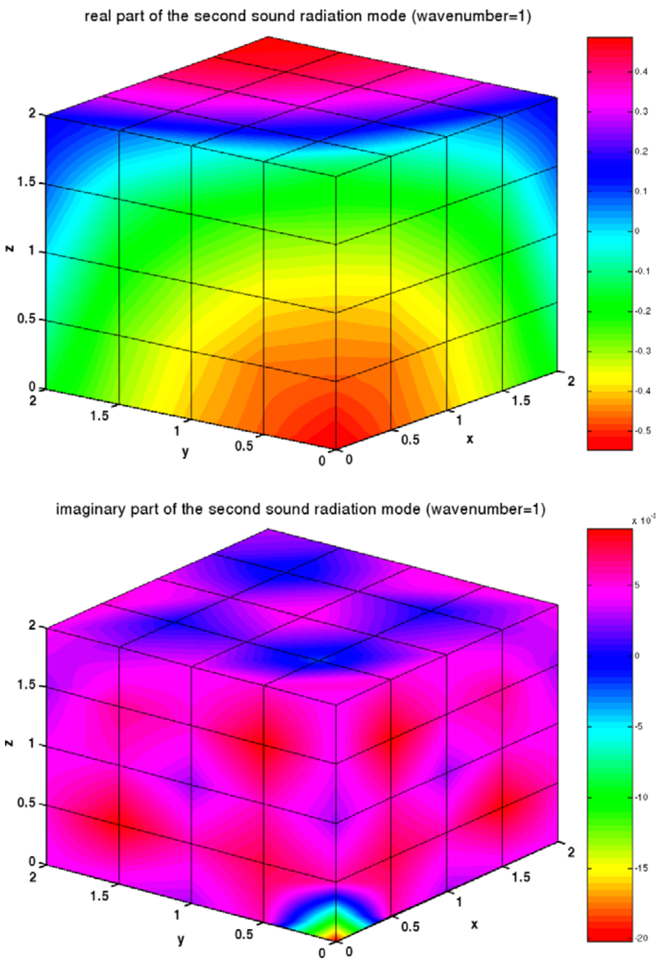

Fig. 10. The second acoustical mode of radiating cube for $k = 1$.

5.2. A radiating cube

The problem of a pulsating cube is formulated by prescribing the normal velocity on a cubical surface produced by a pulsating sphere of radius r_0 circumscribed by the cube, depicted in Fig. 6. The boundary condition prescribed on the cube is given by

$$v_n(r) = \frac{r_0}{r} v(r) = \frac{r_0}{r}\left(-\frac{1}{i\omega\rho}\frac{\partial p(r)}{\partial r}\right) \tag{44}$$

where the sound pressure $p(r)$ can be obtain by Eq. (42). Radius $r_0 = 1$ m, air density $\rho = 1\ \mathrm{kg/m^3}$, sound velocity $c_0 = 1$ m/s, and the length of cube $L = 2$ m, respectively.

Fig. 6 shows the results of radiation cube at selected points with two separate discretization schemes using quadrilateral isoparametric elements. In Table 1, the results at selected points are compared with the theoretical results calculated from Eq. (42).

Fig. 7 gives the former 20 sound radiant efficiencies of acoustical modes with different element sizes when $k = 1$, the results show that the element size has a limit influence on the sound radiant efficiency. As the cube is symmetric in space, the sound radiant efficiencies are also same in three directions. The results also show that sound radiation of cube is concentrated on the former four radiant modes under the condition of low frequency.

Fig. 8 shows the sound radiation efficiencies of former four sound radiant modes. The results show that the sound radiation mainly comes from the first acoustical mode when wavenumber is less than 1. The sound radiation efficiencies have a little difference when wavenumber is more than 1.5. Figs. 9 and 10 plot the former two radiant modes, the results also test that the sound power is concentrated on real part, and the first sound radiant mode is the most efficient one, and the different color indicate the different velocity amplitude distribute of the radiating cube surface. Table 2 lists the sound radiation efficiencies of former 16 sound radiant modes of different wavenumber, the results show that sound efficiencies have a slightly change of former 16 sound radiant modes when wavenumber is more than 3.

Table 2
The former 16 sound efficiencies of sound radiation modes

	$k = 0.5$	$k = 1$	$k = 1.5$	$k = 2$	$k = 2.5$	$k = 3$	$k = 3.5$	$k = 4$	$k = 4.5$	$k = 5$
1	0.3447	0.7539	0.9764	1.3742	1.4280	1.4801	1.6514	1.6495	1.7192	1.9010
2	0.0465	0.4839	0.9741	1.3742	1.4280	1.4757	1.6514	1.6495	1.7046	1.9010
3	0.0464	0.4829	0.9741	1.2085	1.3783	1.4757	1.6514	1.6495	1.7046	1.6665
4	0.0464	0.4829	0.9725	1.2062	1.3731	1.3571	1.4875	1.5199	1.6379	1.6636
5	0.0017	0.0637	0.6340	1.2062	1.3731	1.3571	1.4873	1.5199	1.6379	1.6338
6	0.0016	0.0637	0.6340	1.0988	1.2252	1.2992	1.4873	1.5199	1.6203	1.6338
7	0.0016	0.0473	0.3393	0.8079	1.0991	1.2992	1.4282	1.4999	1.6203	1.6324
8	0.0015	0.0470	0.3381	0.8057	1.0991	1.2992	1.4282	1.4368	1.6203	1.6252
9	0.0015	0.0470	0.3381	0.8057	1.0931	1.2494	1.4282	1.4368	1.5747	1.6252
10	0.0015	0.0039	0.0488	0.3128	0.8920	1.2404	1.3519	1.4307	1.5747	1.6246
11	0.0015	0.0032	0.0408	0.3128	0.8920	1.2404	1.3383	1.4295	1.5614	1.6246
12	0.0015	0.0031	0.0408	0.3097	0.8920	1.1972	1.3352	1.4295	1.5221	1.6214
13	0.0013	0.0031	0.0403	0.2980	0.8631	1.1288	1.3352	1.4233	1.5221	1.5952
14	0.0013	0.0030	0.0403	0.2980	0.8631	1.1288	1.3339	1.3971	1.5221	1.5952
15	0.0012	0.0030	0.0403	0.2980	0.8579	1.1228	1.3339	1.3971	1.5144	1.5952
16	0.0011	0.0030	0.0377	0.2800	0.7000	1.0354	1.2707	1.3717	1.5144	1.5179

6. Conclusion

The method to calculate sound radiant modes and sound radiant efficiency is proposed in this paper via boundary element method and general eigenvalue decomposition. Sound radiation can be decoupled when using radiant modes calculate the sound power. The radiant mode is independent of each others, and the sound power and radiant efficiency can be directly expanded as the sum of the sound radiant modes.

A methodology to solve the sound pressure and sound power by boundary element method presented in this paper. It is should be noted that most of arbitrary structure surfaces are composed by two main kinds of feature geometry with corner and curve face respectively. Accordingly, two structures including the corner and curve face are selected as the calculation examples so as to demonstrate the validity of the method for structures having complex geometry.

The sound radiant mode is independent of the thickness and material property of structures, and just associated with the wavenumber and geometry size. On the condition of the low frequency, the sound radiant power mainly comes from the former 10 radiant modes.

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