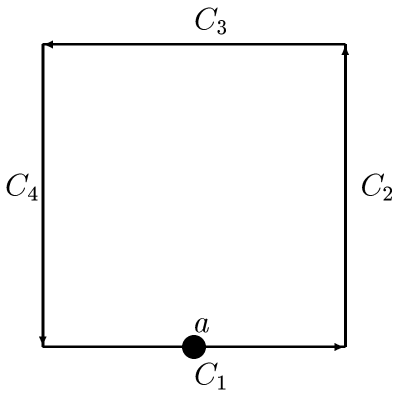
國立臺灣海洋大學河海工程學系1998 工程數學(三)期末考

1. Solve the particular solution (steady state solution) of the SDOF vibration system $\ddot{x}(t) + \omega^2 x(t) = e^{i\bar{\omega}t}$

(a). For the case of $\omega \neq \bar{\omega}$, derive the particular solution $x(t)$. (10%)

(b). For the case of $\omega = \bar{\omega}$, derive the particular solution by the limiting process of $\bar{\omega} \to \omega$? (10 %) (Hint: by superimposing the particular solution with a complementary solution before taking limit)



2. According to the above figure, determine the following integrals. (30 %)

$$A_{11} = C.P.V. \int_{C_1} \frac{1}{z-0.5} dz$$

$$A_{12} = \int_{C_2} \frac{1}{z-0.5} dz$$

$$B_{11} = H.P.V. \int_{C_1} \frac{1}{(z-0.5)^2} dz$$

$$B_{12} = \int_{C_2} \frac{1}{(z-0.5)^2} dz$$

$$A_{12} + A_{13} + A_{14} = \int_{C_2+C_3+C_4} \frac{1}{(z-0.5)} dz$$

$$B_{12} + B_{13} + B_{14} = \int_{C_2+C_3+C_4} \frac{1}{(z-0.5)^2} dz$$

where $C.P.V.$ is Cauchy principal value, $H.P.V.$ is Hadamard principal value, C_1 is the line element from (0,0) to (1,0), C_2 is the line element from (1,0) to (1,1), C_3 is the line element from (1,1) to (0,1), and C_4 is the line element from (0,1) to (0,0).

3. In the class, we have derived the Cauchy-Riemann equations as follows

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Prove that (20 %)

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial t}$$

$$\frac{\partial u}{\partial t} = -\frac{\partial v}{\partial n}$$

where n and t denote the normal and the tangent vectors as follows

$$n = (n_1, n_2)$$

$$t = (-n_2, n_1)$$

(Hint: using the definition of directional derivative)

4. If u and v satisfy the Cauchy-Riemann relations in the region R, then prove $(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) + i(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})$ is analytic in R. (10 %)

5. Calculate the following integral (10 %)

$$\int_0^{\pi} \frac{1}{1 - 2acos(\theta) + a^2} d\theta, \ a > 1$$

6. Given a conformal mapping from (x, y) to (u, v) as

$$w(z) = \frac{z + \frac{1}{4}}{z + 4} = u(x, y) + i\, v(x, y)$$

What is the mapping shape and equation in (x, y) plane of a circle in (u, v) plane of equation $u^2 + v^2 = \frac{1}{4}$? (10 %) What is the mapping shape and equation in uv plane of a circle in (x, y) plane of equation $x^2 + y^2 = 1$? (10 %)

7. Given a conformal mapping from (x, y) to (u, v) as

$$w(z) = z + \frac{1}{z} = u(x, y) + i\, v(x, y)$$

What is the mapping shape and equation in (u, v) plane of a circle in (x, y) plane of equation $x^2 + y^2 = 1$? (10%) How about $x^2 + y^2 = 4$? (10 %)

8. Determine the following integral (20 %)

$$\int_{-\infty}^{\infty} \frac{-1}{z^2} e^{izx} dz$$

海大河工系— 1998 by Chen for complex variable

【存檔：$e:/ctex/course/math3/m3fin97.te$】【建檔:Dec./16/'97】