

國立臺灣海洋大學河海工程學系1997 工程數學 (三) 第十一次作業

1. Determine the following integral

$$\int_C \frac{1}{z^2} dz$$

where C is the line element from (r_1, θ_1) to (r_2, θ_2) in terms of polar coordinate.

(a). Proof by complex method for

$$\int_C \frac{1}{z^2} dz = \frac{1}{r_1} \cos(\theta_1) - \frac{1}{r_2} \cos(\theta_2) + i \left\{ \frac{1}{r_2} \sin(\theta_2) - \frac{1}{r_1} \sin(\theta_1) \right\}$$

(b). Proof by real method using the line element (x_1, y_1) to (x_2, y_2)

$$z = x + yi$$

$$dz = dx + dyi$$

$$x_1 = r_1 \cos(\theta_1)$$

$$y_1 = r_1 \sin(\theta_1)$$

$$x_2 = r_2 \cos(\theta_2)$$

$$y_2 = r_2 \sin(\theta_2)$$

(c). Determine the solution for the special case for the line element $(r_1, 0)$ to $(r_2, 0)$ in terms of polar coordinate.

(d). Determine the solution for the special case for the line element (r_1, π) to (r_2, π) in terms of polar coordinate.

(e). Determine the solution for the special case for the line element (r_1, π) to $(r_2, 0)$ in terms of polar coordinate.

(f). Is the answer frame of indifference ?

海大河工系—1997 by J. T. Chen for complex variable

【存檔：e : /ctex/course/math3/m3hw11.te】 【建檔:Dec./25/'97】