Conformal mapping.

$$z=\frac{w}{1-wa}-C_1$$

where $z=x+iy, w=u+iv$ and C_1 is the center of inner circle from the origin o.

$$x+iy=\frac{u+iv}{1-a(u+iv)}-C_1=-C_1+\frac{u-a(u^2+v^2)}{(1-au)^2+a^2v^2}+i\frac{v}{(1-au)^2+a^2v^2}$$

$$x=-C_1+\frac{u-a(u^2+v^2)}{(1-au)^2+a^2v^2}$$

$$y=\frac{v}{(1-au)^2+a^2v^2}$$

Bipolar coordinate

$$z=x+iy=-C\coth\left(\frac{i(\xi+i\eta)}{2}\right)=\frac{C\sinh(\eta)}{\cosh(\eta)-\cos(\xi)}+i\frac{C\sin(\xi)}{\cosh(\eta)-\cos(\xi)}$$

$$x=\frac{C\sinh(\eta)}{\cosh(\eta)-\cos(\xi)}$$

$$y=\frac{C\sin(\xi)}{\cosh(\eta)-\cos(\xi)}$$

where C is the pole location of the bipolar coordinate system in the x-direction.

Solve the equation

$$x=\frac{C\sinh(\eta)}{\cosh(\eta)-\cos(\xi)}=-C_1+\frac{u-a(u^2+v^2)}{(1-au)^2+a^2v^2}$$

$$y=\frac{C\sin(\xi)}{\cosh(\eta)-\cos(\xi)}=\frac{v}{(1-au)^2+a^2v^2}$$

we obtain

$$u=\frac{-(-aC^2+C_1+aC_1^{\ 2})\cos(\xi)+(C_1+a(C^2+C_1^{\ 2}))\cosh(\eta)+C(1+2aC_1)\sinh(\eta)}{(-1+aC-aC_1)(1+a(C+C_1))\cos(\xi)+(1+2aC_1+a^2(C^2+C_1^{\ 2}))\cosh(\eta)+2aC(1+aC_1)\sinh(\eta)}$$

$$v=\frac{C\sin(\xi)}{(-1+aC-aC_1)(1+a(C+C_1))\cos(\xi)+(1+2aC_1+a^2(C^2+C_1^{\ 2}))\cosh(\eta)+2aC(1+aC_1)\sinh(\eta)}$$