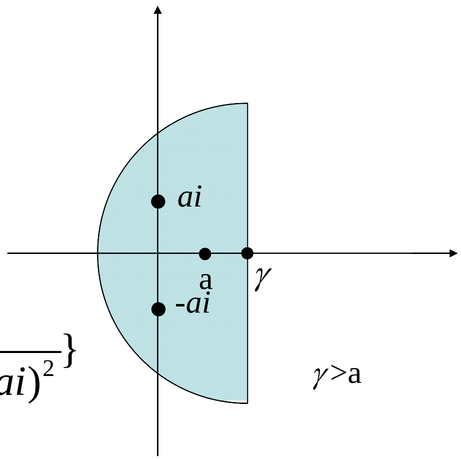
Inverse Laplace Transform II

(1) $e^{at} \rightarrow \dfrac{s}{(s^2+a^2)^2}$

Inverse Laplace transform： $\dfrac{1}{2\pi i}\int_{\gamma\text{-}i\infty}^{\gamma+i\infty} F(s)e^{st}ds, \gamma > 0$

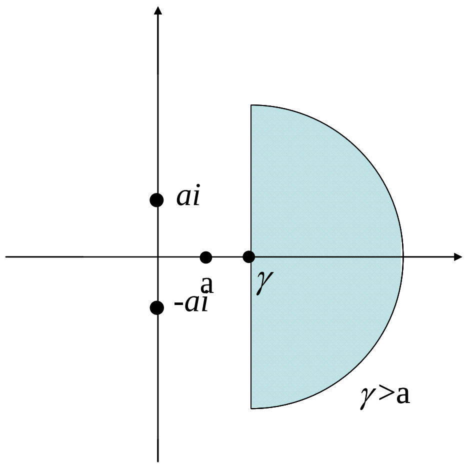
$$\frac{1}{2\pi i}\oint F(s)e^{st}ds = \frac{1}{2\pi i}\oint F(z)e^{zt}dz$$



$$F(s)=\frac{s}{(s^2+a^2)^2} = \frac{1}{4ai}\{\frac{1}{(s-ai)^2} - \frac{1}{(s+ai)^2}\}$$

$$\frac{1}{2\pi i}\oint F(s)e^{st}ds = \frac{1}{2\pi i}2\pi i\{\frac{1}{4ai}te^{st}\Big|_{s=ai} - \frac{1}{4ai}te^{st}\Big|_{s=-ai}\} = \frac{t}{2a}\sin(at)$$

$$\frac{1}{2\pi i}\oint F(s)e^{st}ds = 0,\ t < 0$$



Note: decompose 3 methods:
Fractional formula and Taylor expansion