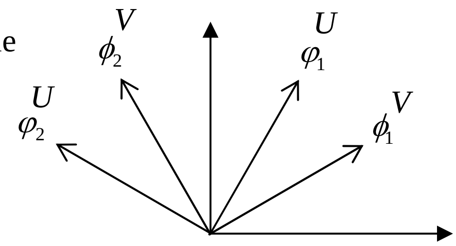
SVD Complex 操作

$F = \Phi\Sigma\Psi^T = RU = VR$ Matrix

$Fx = e^{i\phi}\mathscr{T}(e^{i\Psi}z)$ Complex variable



where

$$\mathscr{T}(z) = \frac{1}{2}(\sigma_1 + \sigma_2)z + \frac{1}{2}(\sigma_1 - \sigma_2)\bar{z}$$

σ_1, σ_2 are singular values

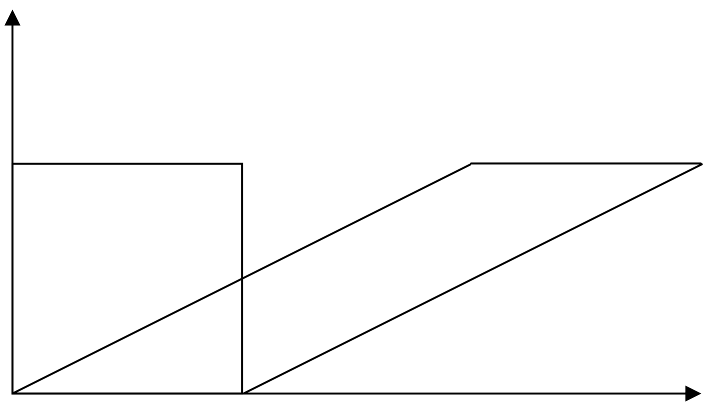
$$\Psi^T = \begin{bmatrix} \cos(\Psi) & -\sin(\Psi) \\ \sin(\Psi) & \cos(\Psi) \end{bmatrix} \quad \Phi = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & \frac{2}{\sqrt{3}} \\ 0 & 1 \end{bmatrix} \quad \Phi = \begin{bmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{bmatrix}$$

$$U = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{5}{2\sqrt{3}} \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{5\sqrt{3}}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad \Psi^T \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) \\ \sin(-60^\circ) & \cos(-60^\circ) \end{bmatrix}$$

$$R = \begin{bmatrix} \cos(-30^\circ) & -\sin(-30^\circ) \\ \sin(-30^\circ) & \cos(-30^\circ) \end{bmatrix}$$



$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \Phi\Sigma\Psi^T \begin{Bmatrix} x \\ y \end{Bmatrix}$$

$$u + vi = e^{i\phi}\mathscr{T}(e^{i\varphi}z) = \frac{1}{2}(\sigma_1 + \sigma_2)e^{i(\phi+\varphi)}z + \frac{1}{2}(\sigma_1 - \sigma_2)e^{i(\phi-\varphi)}\bar{z}, \text{ where } z = x + yi$$

Ref:

H. C. Wu, Continuum mechanics and Plasticity, CRC Press, 2005.

J. T. Chen, C. F. Lee and S. Y. Lin, 2002, A new point of view for the polar decomposition using singular value decomposition, Int. J. Comp. Numer. Anal. Appl., Vol.2, No.3, pp.257-264. ***(NSC-90-2011-E-019-006)***