系級:______________ 姓名:______________ 學號:______________ 考試範圍-Series

國立臺灣海洋大學河海工程學系 2001 工程數學 (三) 第一次作業小考

1. Solve the ODE using series form:

$$(1-x^2)y''(x) - 2xy' + 2y = 0,$$

$$(1-x^2)y''(x) - 2xy' + 6y = 0,$$

$$(1-x^2)y''(x) - 2xy' + 12y = 0,$$

$$(1-x^2)y''(x) - 2xy' + N(N+1)y = 0,$$

(a). $y(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$

Please determine the relations for the coefficients, $c_i, i = 1, 2, 3, \cdots$
Please determine the Lengendre polynomials.

2. Find the Wronskian for the two complementary solutions. If the Lengendre polynomial is known in priori, solve the other complementary solution using Wronskian approach.

3. Find another complementary solution by using variation of parameters.

$$(1-x^2)y''(x) - 2xy' + N(N+1)y = 0,$$

$$y_n(x) = P_n(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots c_n x^n$$

$$y_2(x) = P_n(x)u(x)$$

4. Solve the ODE using series form:

$$-x^2 y''(x) - 2xy' + n(n+1)y = 0,$$

$$y_n(x) = x^n \text{ or, } \frac{1}{x^{n+1}}$$

$$(1-x^2)y''(x) - 2xy' + n(n+1)y = 0,$$

$$y_n(x) = P_n(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots c_n x^n$$