系級:_____ 姓名:_____ 學號:_____

國立臺灣海洋大學河海工程學系 2001 工程數學 (三) 第二次作業小考

兩階線性微分方程系統性解法

1. Step 1: given one complementary solution, y_1.

2. Step 2: solve another complementary solution $y_2 = y_1 u_1$.

3. Step 3: solve another particular solution $y_p = y_1 v_1 + y_2 v_2$.

4. Example :

$$x^2 y''(x) - 4xy' - 6y = -6 \tag{1}$$

(a). Assume the $y = x^n$ for the complementary solution, determine n. (5%)
(b). If $y_1(x) = \frac{1}{x}$ is one of the complementary solution, please determine the other one $y_2(x)$ by method of variations of parameters, $y_2(x) = y_1(x)u_1(x)$. Please find $u_1(x)$. (5%)
(c). Solve the particular solution by $y_p(x) = y_1(x)v_1(x) + y_2(x)v_2(x)$, where

$$y_1 v_1' + y_2 v_2' = 0$$

$$y_1' v_1' + y_2' v_2' = \frac{-6}{x^2}$$

Please determine v_1, v_2 and y_p. (5%)
(d). By changing variable, $x = e^t$ and $y(x) = y(e^t) = Y(t)$, then determine the ODE for $Y(t)$ and solve $Y(t)$ and $y(x)$. (5%)
(e). By taking the Laplace transform twice with respect to Eq.(3), derive the results. (5%)