國立臺灣海洋大學河海工程學系 2001 工程數學 (三) 第五次作業小考

1. The origin is an ordinary point of the Chebyshev equation,

$$(1-z^2)y'' - zy' + m^2 y = 0,$$

which therefore has series solutions of the form $z^\sigma \sum_0^\infty a_n z^n$ for $\sigma = 0$ and $\sigma = 1$.

(a)Find the recurrence relationships for the a_n in the two cases and show that there exist polynomial solutions $T_m(z)$:
(i) for $\sigma = 0$, when m is an even integer, the polynomial having $\frac{1}{2}(m+2)$terms;
(ii) for $\sigma = 1$, when m is an odd integer, the polynomial having $\frac{1}{2}(m+1)$terms.

(b)$T_m(z)$ is normalized so as to have $T_m(1) = 1$. Find explicit forms for $T_m(z)$ for $m = 0,\ 1,\ 2,\ 3$.

(c)Show that the corresponding non-terminating series solutions $S_m(z)$ have as their first few terms

$$S_0(z) = a_0(z + \tfrac{1}{3!}z^3 + \tfrac{9}{5!}z^5 + \cdots),$$

$$S_1(z) = a_0(1 - \tfrac{1}{2!}z^2 - \tfrac{3}{4!}z^4 - \cdots),$$

$$S_2(z) = a_0(z - \tfrac{3}{3!}z^3 - \tfrac{15}{5!}z^5 - \cdots),$$

$$S_3(z) = a_0(1 - \tfrac{9}{2!}z^2 + \tfrac{45}{4!}z^4 + \cdots).$$